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30.05.2009: Informatics extended. #3
24.05.2009: Mathematics and set theory fundamentally reworked. #2
25.04.2009: Pages migrated to XHTML 1.1, search function improved and used files more uniformly renamed.
22.02.2009: Subject catalogue improved and homepage from HTML migrated to XHTML.
11.01.2009: Set theory amended and number theory inserted. #1
05.12.2008: Size- and colour change and also RSS feed inserted.
09.11.2008: Set theory improved.
24.08.2008: Linear programming inserted.
05.07.2008: Internet and co extended.
04.05.2008: Introduction and politics completed.
03.05.2008: Definitions and glossary inserted.
29.04.2008: Error site set up and language independent code programmed.
13.04.2008: Homepage after W3C validated and PHP code as text inserted.
05.04.2008: Informatics reworked and internet and co and sorting newly inserted.
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26.10.2007: Bibliography inserted and introduction completed.

16.09.2007: Computation of time completed.
11.02.2007: Numbers/set theory shortened to set theory.
29.01.2007: Physics inserted.
23.01.2007: Numbers/set theory extended.
22.01.2007: Numbers/set theory completed.
07.01.2007: Numbers/set theory changed.
02.01.2007: Numbers/set theory completed by more examples.
01.01.2007: Numbers/set theory extended by considerations concerning the approximation order.
31.12.2006: Mathematics completed.
30.12.2006: Informatics completed with considerations to the n-dimensional space.
05.11.2006: About me revised.
03.09.2006: Complete revision of the sites.
30.07.2006: Private sector with Web log as diary set up.

Current

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02.07.2019: Publishing the seventh edition of "Nonstandard Mathematics".
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02.07.2017: Publishing the fifth edition of "Nonstandard Mathematics".
01.07.2016: Publishing the fourth edition of "Nonstandard Mathematics".
21.06.2015: Publishing the third edition of "Nonstandard Mathematics".
20.12.2014: Publishing the second edition of "Nonstandard Mathematics".
17.08.2014: Appearance of the publication "Nonstandard Mathematics".
02.06.2013: Riemann zeta function determined for all natural arguments.
24.02.2013: Strongly polynomial normal method presented.
14.01.2013: Terms of openness and closure of sets refuted.
25.11.2012: Proposition concerning the simplex method proved.
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20.10.2011: Publication of the e-book "Relil - Religion und Lebensweg".
25.09.2011: Presenting the idea of radio computing.
13.02.2011: Presentation of the programming concept Stair.
12.11.2010: Challenge of several axioms of the Euclidean geometry.
30.05.2010: Definition of the complex and real numbers.
11.04.2010: Elucidation of fast indexing and of searching in O(1).
21.03.2010: Presentation of the sorting procedure immediate sort.
23.11.2009: Presentation of two models of the universe.
24.05.2009: Numbers of algebraic numbers now proved.
11.01.2009: Sensational breakthrough in the field of transcendental numbers achieved.
01.12.2007: I convert the homepage into PHP and MySQL.

Index of Abbreviations

AD ..... antiderivative
AN ..... algebraic number
CP ..... closed path
DFT ..... discrete Fourier transform
GPC ..... greatest-prime criterion
GS ..... geometric series
LI ..... line integral
LP ..... linear programme
NR ..... neighbourhood relation
TS ..... Taylor series

Introduction

What is the aim of this homepage?

This homepage is result of my independent considerations in informatics, mathematics, music and physics. The ideas come almost assumedly from God, what I reciprocated extraordinarily thankfully with different actions. The attained thoughts and results of research shall address a circle as wide as possible. The individual web pages are mostly rather skimped.

Citations from other resources are widely missing. If an essential consideration should have existed before this publication, I will insert corresponding citations according to a message to me. But a bibliography is specified that includes almost completely literature in my possession, in order to reveal resources helpful for understanding and to offer opportunities for additional information.

The informatics part introduces the fast sorting method bit sort below theoretical informatics. The halting problem is newly answered with a positive respond and the P versus NP problem is solved. Knowledge of informatics studies is helpful. Radio computing enables, by simultaneous radioing on different frequencies, parallel computing through parallel intercellular communication.

Programming with Pfeil presents a future fast, multi-dimensional and relational programming language that is intended to make programmers' work significantly easier by using bots to search the Internet. Internet and Co outlines what role the internet could play in the future, for what the multifunction device strap could serve and what could hold for transport and machines.

Mathematics treats Euclidean geometry, equitable distributing, nonstandard analysis, linear programming, infinite sets, topology, number theory and computation of time. The set theory clarifies how many elements important (in)finite sets have. The nonstandard analysis offers new definitions and (revised) theorems to obtain more precise and better mathematical statements.

It solves the measure problem. In topology, the (simple) connectedness of sets is redefined. The Euclidean geometry challenges several axioms and theorems. Linear programming proves the diameter theorem for polytopes and confronts the simplex method including a new perturbation method with the mostly polynomial intex method. The advice includes results from my mathematical experience.

The number theory specifies necessary and sufficient criteria as well as new examples for transcendental numbers. It deals with Littlewood conjecture and generalised Riemann hypothesis. The computation of time presents a calendar based on the octal system and gives examples for its application to length specifications. Some knowledge of mathematics studies is required to understand several coherencies.

The music can be found on https://en.boris-haase.de/mus.

Physics considers theoretically or experimentally two models of the universe and, in the context of youth research 1983, the period times of selected oscillations of mass points in the plane, for which approximation formulas are specified on the basis of radius of curvature, initial height and backwards pushing force, without integration. To understand, a study of mathematics or physics is helpful.

The sitemap contains also a revisional history; the definitions and the glossary list only terms that are new or are used differently than usual. And now much pleasure with my homepage!

(It does not depend on the pictures and animations: Peace shall emanate from the texts.)

Questions and answers

What does the tightened prime number theorem imply?

It solves the many problems that arise from the invalid (general) Riemann Hypothesis.

What about the solution to the measure problem?

Any set becomes measurable. Measure theory can do without σ-algebras and zero sets. It is simpler overall.

What does the centre method do?

It solves linear inequalities and calculates eigenvalues and vectors in quadratic time.

What about the solution of the P-NP problem?

All (finite) non-deterministic problems can be solved deterministically.

What follows from open and closed being equal?

The topology becomes simpler: both represent redundant terms.

What are the advantages of mid-finite numbers?

They continuously fill the abrupt gap from finite to infinite numbers.

What about the redefinition of transcendental numbers?

It allows limiting to (infinitely) rational numbers and simplifying the theory.

What special role do the natural numbers play?

They are the basis for counting the elements of all existing sets.

What effect have the new operators pre and post?

They make it possible to work with irregular distances in analysis and they greatly simplify it.

What role does the exact integral play?

It is simpler, more precise and more general than the previous integral definitions.

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Informatics

Internet and Co deals with some aspects of the future internet: How can examinations be conducted with the aid of the internet? How can a (number of) identification, the ID, assigned uniquely to each person, usefully be applied in connection with the internet? What types of content will be on the internet? How can information user profiles simplify search processes on the internet?

What tasks could a multifunction device called strap fulfil? What can modern clothes achieve (for deafblind people)? How can virtual reality be realised (in rooms and in the brain)? How can goods be transported quickly in future? What are the consequences for professions in the future? What must be in future considered for intelligent machines?

Programming with Pfeil presents a future fast, multi-dimensional and relational programming language that is intended to make programmers' work significantly easier by using bots to search the Internet and convert programs into other programming languages that are particularly suitable for the individual problem, and automate and certify them proven and optimised.

The theoretical informatics presents a $\mathcal{O}(1)$ sorting algorithm for non-bounded hardware. The organisation of the memory locations as a binary tree makes it possible to use the $\mathcal{O}(n)$ method bit sort for n input values read in succession. The minimum or maximum framework of a graph with n nodes is calculated in $\mathcal{O}(\log n)$. The halting problem newly answered with a positive respond and the P versus NP problem is solved.

Radio computing enables, by simultaneous radioing on different frequencies, parallel computing through parallel communication of each memory cell with each other one. Thus, n values can be sorted in $\mathcal{O}(1)$, the indexing of database tables becomes widely obsolete. Networking and cloud computing solve with radio technology various problems much faster than before, what can be accessed comfortably at home.

All problems of an arbitrary (higher) world can be solved at one go by singling out those from all possible solutions with recording the solution formations by (comparative) sorting that satisfy the solution criteria. The gödelisation renders substantial comparisons broadly unnecessary. Consequently, all problems are for L in the same complexity class $\mathcal{O}(1)$.

L does not get around the formation of all possibilities, since crucial problems are irreducible in terms of complexity. For our (finite) world, this means that crucial problems are not solvable in it. This does not mean, however, that we cannot solve the problems important for us, but only, that we cannot answer all scientific questions.

Theorem: All problems of a finite world can be solved in $\mathcal{O}(1)$.

Proof: Conditions are checked by inserting the complete solution space into the conditions at once, possibly by parallel processing. Calculations are accelerated by summarising calculation steps. Continued summarising yields an overall calculation step. Hence the claim follows, since every finite solution space can also be constructed in one step.

Radio Computing

How can a memory cell communicate with as many other ones by radio chips at different frequencies in order to support parallel computing? How can in this way sorting of n values be improved from $\mathcal{O}(n \log n)$ to $\mathcal{O}(1)$ and thus the indexing of tables in databases become obsolete?

If each radio chip transmits (simultaneously) at the frequency that corresponds to the value, transformed into the smallest positive natural interval valid for all values, of the associated memory cell, then the counterpart stations receive these new values sorted in $\mathcal{O}(1)$ and they can output them in $\mathcal{O}(1)$, if they are hierarchically organised as a binary tree, while first the 0-node and then the 1-node sends to its parent node.

If the parent node is still not taken, it stores the value of the 0-node. For each free further parent node, the procedure is the same and the 1-node is pulled behind. This is altogether possible in $\mathcal{O}(\log n)$, thus enabling an output of all nodes by traversing the resulting binary tree in $\mathcal{O}(n)$. Equal values are first made different by appending a sequence number in $\mathcal{O}(n)$.

It is clear that the radio distances depend only on the size of the radio chips and that the radio must not be disturbed from the outside. Here weak (if need be shielded) signals and a predetermined frequency range are advisable. Possibly, first an implementation in datacentres is provided, as long as the radio chips are still relatively large, but this is for example not a big problem because of the increasing cloud computing.

For fewer frequencies than values to be transmitted, use FSO or microwaves, or double the number of transmission runs for each additional bit. If the frequency values are smaller than the values to be transmitted, so another transmission run for the subsequent sorting is necessary, for every transcending by a further frequency range. If applicable, for additionally occurring values, overflow areas are advisable (e.g. for table queries).

If we arrange the radio chips in a straight line and put in each case two of these lines crosswise one above the other, they can, if they are confronted with, send without intersection, and we get along with one or few frequencies. Thus, $k + 1$ tables can be joined in a query in $\mathcal{O}(1)$ instead of $\mathcal{O}(n \log_k n)$. However, the necessity for serial processing sets limits to the parallel (radio) computing.

The radio technology can also be used to implement parallel transmission of parallel adders or other (linear) arithmetic operations (e.g. comparisons). In conjunction with networks and fast data transmission (e.g. DSL), it can be parallel calculated in combination (e.g. to solve (linear) optimisation problems in linear time). Associative problems can be solved in parallel (maybe better than in the brain), too.

Internet and Co

Exams

Ensuring the authenticity of information requires appropriate safety mechanisms - such as for exams. Authentication can be done using biometric methods. Supervised computers are suitable as the only foreseeable inexpensive solutions, since both humans and computers can implant unwanted information.

The supervised computers must be able to identify, store and display for usage the brought along allowed additional information as such and to specifically prevent or channel access to the Internet. For this purpose, both appropriate standards and intelligent verification methods have to be developed if we want to achieve a certain level of comfort.

An easy way to do this is to specify the allowed URLs for the browser. Automatic methods for image recognition that can filter out undesirable sites are not suitable because possibly encrypted sites can only be decrypted with great effort. Thus, only sites are in question, which have a fixed incorruptible authorship and no hidden areas.

Reliable control can only be guaranteed if it is impossible to manipulate the assignment of URL and content (be careful with generic URLs!). Suitable passwords can protect additional information that has been brought along and decrypted before the exam (on the Internet). Customising the programmes for additional information must not take up too much time.

For this purpose, password-protected profiles of the programmes should be safely stored, which then only have to be read in (requires a corresponding implementation of the programmes). If the Internet should develop to the extent that every programme is available there, this problem is omitted. Since special knowledge is usually also to be checked by special programmes, it is something desirable.

Programmes that cannot be called up via the Internet may possibly be called up via appropriate controlled remote connections, whereby speed is no problem. It becomes more difficult when the exam situation requires special hardware (e.g. in surgery). There will always be exams tied to a specific location. Most exams can yet be realised on standard devices.

If knowledge from certain time units is in demand, a questionnaire is appropriate that, varied at given times, the examinees have to answer at the same time. For the success, a certain percentage of correct answers for individual fields may be demanded, which can vary depending on the field. Multiple-choice questions facilitate the evaluation on the computer, as long as the automatic language comprehension is limited.

For exams via internet, the automatically evaluated result can be announced immediately after the end of the exam. At some point, the exam possibilities and fairness require the Internet: No one can be expected to keep all the necessary knowledge in mind. The irreplaceable achievement of man is in the creative and information connecting field.

The protection of the examinee's personality may require anonymised exams on the Internet. For this purpose, the testing centre should offer signatures on the Internet with which the examinee can subsequently be authenticated. Furthermore, the testing centre should offer anonymous (temporary) email accounts to view the exam result by the examinee using signature and password.

Email

Voting on the Internet is also easily possible if the identity of the voter can be uniquely clarified. Everybody should be assigned a free email account at birth, to which all emails are sent that require a unique authentication. This email account would even have a higher status than the letterbox concerning security aspects.

It could be (co-) administered by the legal guardians until early infancy. For this purpose, every person would need to have Internet access if necessary (possibly in public authorities). There could be helped all those with the handling, which have problems with it. In the future, the handling must be possible voice-controlled without the need for a keyboard.

Each person should also receive at birth a globally unique (number of) identification that can be pronounced as a name (see Linguistics) and officially serves as such. This ID also serves as a (method) signature of important documents. Its clear generation from immutable biometric data is worth considering. Easier is the query in a secure database.

The consistent use of the ID allows the fast and reliable retrieval of specific content about oneself on the Internet. With increasing improvement of spam filters, the email account assigned at birth can be unambiguously and for each person ascertainable linked to the ID: e.g. [ID]@mail.world. Everybody should have free webspace available.

Contents

The latter should, for example, be accessible at www.[ID].net.world and free of unwanted content. This is watched over by administrators appointed by the future world government. Each presentation should also be available in the future world language for reasons of comprehensibility. For the official part, an obligatory profile can be prescribed. It may be linked to unobjectionable home pages.

People who do not want to post own content are represented by minimal content as e. g. can be found in an identity card. All people have to be informed about their stored data and their location. When creating their own content, people are helped by the authorities as much as possible. The ascertainability by a uniform questionnaire determines the content that can be claimed.

A law that is valid worldwide determines the volume of the minimum content. For example, a restriction on the publication of ID and nationality is conceivable. Given that nationality will play a less and less important role in the face of global civil society, it is quite conceivable that the law only considers the ID to be mandatorily published. We only know then that this ID exists, but not anymore.

The principle of equality has no bite here, since each person may determine the extent of the own representation in public. The authorities may save significantly more data concerning an ID to discharge their duties. People can only demand within the valid laws to view the data stored about them. Sufficient data privacy has always to be guaranteed.

Anyone publishing data on www.[ID].net.world explicitly consents to their use for all legal purposes. This includes in particular the automated processing. All people have to be informed about what can be done theoretically and practically with their data. Thus, they get a comprehensive idea and can realise the scope of their decisions.

The right to free self-expression includes that the published data need not be truthful. This should be clear to everybody. No one can demand a correction. No one can also demand that somebody corrects the truthful data published elsewhere if it does not violate the personal rights of those described.

Public persons can defend themselves to a lesser degree against depictions of themselves, as the legitimate interest of the public can conflict with their personal rights. However, a core of personal rights remains invulnerable, as human dignity must be preserved. More details are regulated by the laws of the world government and should not be subject here.

Persons may encounter untrue allegations of facts about them with counter statements. The latter have also to be published on the Internet regardless of their truth. It does not matter how often the homepage is viewed. If the representation is sufficiently corrected or withdrawn, the right to publish the counter-statements expires.

The freedom of publication allows that publications about described persons have not to be displayed to these persons, since the latter can easily search the Internet for their person by means of their ID. If these persons would like to be informed about them, then this wish should be met by mailing to their desired email address. That's no effort!

Internet reception should also be possible on a mobile basis: via satellites or sufficiently dense distributed hotspots or similar. Internet control should be possible via voice (headset), data gloves or eye movements.

The monitor is replaced by data glasses (with prompter) switched semi-transparent with possibly different content for each eye - even with 3D representations of the same content.

In future, we can arrange radio broadcasts for us or others on the Internet on our own. Newscasts can be specifically read to us (by a female, male or child's voice). Here, the voice (such as the spouse) can be selected specifically. This also applies to musical pieces or pools. The costs are like those of radio or television.

Foreign programmes can be combined with own broadcasts (live conferences with microphone or webcam). Examples are own concerts or play stagings, art exhibitions with virtual tours and explanations, (collective) karaoke, commercial spots for own products, workshops or conference circuits. The future is multimedia almost everywhere.

Automatic simultaneous translations via Internet become imaginable with the increasing development of speech recognition and translation programmes if the introduction of the worldwide uniform planned language is delayed. The smarter the programmes are, the more likely they can correct errors in speaking, and, if necessary, interpose explanations. It remains reserved to the internet users what they allow or initiate.

In order to simplify the search processes, the existing idea of crawlers is extended to (standardised) information user profiles. The latter specify user requests for a site, which then each page can respond by appropriate queries such that the desired content is almost exclusively displayed. For this purpose, the programming language PHP, for example, or the browser could be extended.

Categories would have smartly to be formed, which could be queried. The browser should be able to summarise the content that is scattered across several sites on one site to obtain a clear representation. Ontology concepts should be meaningfully included (e.g. through query rules formulated on terms, which browser and PHP then evaluate).

The advantages of this approach are the free determination of categories that have not to occur in the site and can consist of whole sentences and of possibly complex rules, without significantly extending the search time. Since queries are based on searching in sorted data, the latter may have to be stored as suggested by Theoretical Informatics and Reference Theory.

Strap, clothes and virtual reality

Portable computers may be connected to the so-called strap, which serves as a multifunction device for authentication, is worn like a watch and takes over functions including display, telephone and key function. Almost all functions of the strap can be activated or deactivated at any time by voice or touch.

It also works without a computer connection and replaces an implanted chip. When it is laid down, it triggers an alarm after a predefinable period of time if it is not activated again in a certain way. Certain functions should always be disabled for security reasons if the strap is laid down. The alarm can have the integrated transmitter send messages to security forces.

We can also adjust the strap via black box so that the events of the last few hours around us are recorded and can be sent out when an alarm occurs, or we request that. In order that criminals have no possibility to stop the transfer process, a running and sufficiently quick (encrypted) transfer to a secure computer is suitable.

Since our coordinates are also transferred, the last abode before a possible kidnapping is always known. Versions of the strap may also be used as an electronic ankle monitor. We should consider carefully which functions of the strap are enabled or disabled, since many conclusions regarding our behaviour are possible – if the transfer may be still so secure.

Modern clothing can also be used to convey information: the body surface can absorb patterns of different pressures or different temperatures. Thus, images and language become easily transferable to deafblind. If the garment is connected to a rangefinder or corresponding image recognition software, even blindman's stick can be omitted.

The connection of the garment with other measuring devices allows the inconspicuous evaluation of pollutant concentrations, magnetic fields and everything else, which can be measured as a scaled substance or can be evaluated by programmes (e.g. fluctuations in the voice or the skin or the eye contact of the opposite, which can give indications of certain emotional states).

If the garment is programmable (input via voice, touch or evaluation of the viewing direction), the measured variables can be easily changed, whatever we want to measure. A connection to the Internet and databases increases the range of information once again. However, prices also will rise with the requirements - and decrease with a high number of units again.

The path to virtual reality can lead in rooms (CAVE) not only to projection methods onto suitable coated surfaces but also to programmable and controllable (as above) wallpapers. In this case, a computer determines the image to be projected for existing objects in the space (distortion compensation). Even 3D effects can be achieved (higher dimensions as a projection).

The surfaces of the room can be segmented for different content and can be individually addressed. However, the limited ability of humans to parallel processing sets limits. As long as they are not cyborgs, they remain dependent on computer assistance. Direct influence on the brain is therefore so attractive because impressions have not to be communicated materially (sensory organs).

Transport and machines

For the fast and reliable delivery of goods from warehouses and collection points, partly underground express connections are suitable. The volume of goods is at most 0.125 cubic metres $(0.5\ m)^3$, larger goods may need to be transported more slowly. Computers calculate optimal network utilisation and collision avoidance. They inform the technical staff about occurred jams.

Transport mode could be for example magnetic levitation technology in combination with vacuum tunnels. The goods may be collected and posted at easily accessible shipping points. A transport (with a different technology) to the households is conceivable. Food can be, for example, centrally prepared at the collecting points. By centralising also unusual wishes can be fulfilled.

The individual containers for transport can be streamlined. Integrated chips control the unmanned system. Only a few minutes pass by between order and delivery. The transport fees amount a fraction of the value of the goods if the containers are filled well: driving should be more expensive. Robots or computers will undertake many occupations that we know today.

Man will be still in demand, where complicated conversations with people arise, where creative performance, years of experience and special skills are needed and where complex organisation and coordination is required. Physical work and everything, for what rules can be laid down, is vulnerable to replacement. Our world will be managing with significantly fewer people in the face of rising needs.

Human beings and machines become more and more similar until they merge. They will die if they want to: to enter the subsequent world - at a point they have chosen. Fatal accidents will continue to occur - but less often. The question for the domination through the machines will arise: for this, however, man will have to find a solution or will perish.

One solution is control and monitoring. Programmes have to constantly check the information in a machine. Certain functions in a machine may not be disabled without disabling the machine itself. This involves an interdiction of coding in certain areas. Alternatively, a coding of certain programmes must be able to be decoded simultaneously.

Machines need a central, unforgeable and regularly renewed approval that can be checked at any time - such as the overall status of the machine. Simpler checks must be possible by owners and police, more complex ones may be reserved for specialists. Laws against crime by machines are to be issued and the liability is to be clarified.

Programming with Pfeil

The future programming language Pfeil (German for arrow) is presented here, the name of which refers on the one hand to its relational foundations and on the other hand to its speed ("as fast as an arrow"). It uses most modern concepts, because it is multilingual (the code words are compared to an English specification, for example), Unicode-capable, visual (relationships are represented in multidimensional space that cannot necessarily be easily linearised), declarative and functional (AI is used for code optimisation).

The implementation used for the respective programming project depends on the individual problem and does not (!) need to be specifically defined. Pfeil is also able to generate both a quick and not necessarily identical elegant implementation from the problem for presentation purposes. Translations into any programming language that can be implemented for the respective problem are possible automatically, including the selected notation.

Pfeil is integrated into a development environment that has bots swarming the Internet to create particularly efficient programmes when they are approved accordingly. These are checked as far as possible according to the programmer's specifications, whereby programmer may request a specific certification or state that paid programmes are also used, because e. g. certain quality requirements (of the customer or the law) must be met.

To this end, certain standards are introduced that (Pfeil) programmes must meet to avoid damage. So e. g. a demand is uniquely to clarify the question of liability, since an automatism can be held accountable with difficulty and the evidence can be difficult from a certain program complexity. Arrow can be used (possibly by adding a timeline and AI) both as a markup language (for documentation purposes) and as a real programming language.

Automatic proof facilitates the implementation in other programming languages and enables results that are valid in computer science or mathematics, too. The traceability is given by the breakdown to the simplest connections. The use of AI enables optimised code sections to be saved, the parameters of which can be determined by machine learning. Pfeil allows distributed computing by individually developing and using resources.

The link between different entities is the arrow expressing a relationship: $m{:}n$ relationships can not only be noted easily and elegantly, but also implemented efficiently. Pfeil is a programming language that saves work by not burdening programmers with details that they may not understand but result from machine learning, for example. They may optimise it, but the computers used for Pfeil are too much effective for them.

Pfeil gives programmers useful tips on what their programming should look like by automatically searching for and suggesting comparable problems. The relationships allow a computerised representation of the visual space, which enables an increasingly complex machine understanding. Quantum programming concepts can also be implemented as a matter of course, since Pfeil can handle all hardware implementations that are not too rare.

Pfeil is particularly good at dealing with multi-path processes: optimised sequences are calculated from a simple input, where programmers do not have to worry about the details but can see from the output generated by Pfeil to what extent their programmes are correct or where there is still improvement necessary. Pfeil could only know this if programmers' intentions were clear. Examples here are network flows, optimisation, and transport problems.

Humans cannot easily implement parallel programming, since there are so many conditions to be considered here that it is usually only possible for outspoken specialists. Pfeil, on the other hand, can fall back on ready-made frameworks, which it adapts with the aid of AI. The focus here is less on simple than on complex models that a person can hardly understand. Here, too, reference must clearly be made to the moral integrity of the people, because otherwise there could be many disadvantages.

Discussion: Pfeil is necessary insofar as the world is becoming more and more complex and its connections exceed human comprehension and imagination. Furthermore, the problems are getting bigger and can no longer be solved by individuals. Overall, the solution process is becoming increasingly

unsatisfactory for humans and makes the world less and less worth living in. From an optimistic point of view, Pfeil remains one of the few ways out of the previous disaster for mankind.

Supercomputer

What properties should a supercomputer have that is particularly fast? How should its memory be structured? To what extent does it differ from conventional computer architectures? How can this be illustrated using the examples of basic arithmetic operations and matrix multiplication?

In the following, a bit length of 64 is used for calculation. In practice, however, this can be higher. The basis are i. a. particularly fast counting lines that are used bit by bit for addition. A counting line is run through e. g. of particles such as photons or electrons in a suitable medium, which is not important here, since only the theoretical concept is described. These count the number of set bits of equal value from several adjacent binary numbers.

The decisive factor is the high speed of the particles. The 64 numbers per bit are again stored as binary numbers and added (bitwise shifted). Since this happens in parallel, an addition speed of 2^{32} binary numbers with bit length 32 is achieved in $\mathcal{O}(1)$. Subtractions are implemented using the two's complement. Continued addition enables multiplication and, via forming the inverse, division. A hierarchical memory enables quick comparisons (see Radio Computing).

For fast multiplication of two wlog. square matrices, each with 2^n entries for $n \in {}^{\nu}\mathbb{N}$, these are written into a memory ball by simultaneous replication (i. e. simultaneous sending and storing). The multiplication in $\mathcal{O}(1)$ takes place in its nodes, in which both entries of one matrix are stored. Then the entries of a row are added in parallel in $\mathcal{O}(1)$ as described above via counting lines, which calculates the result matrix.

Here, the root is in the innermost shell, its leaves in the next outer shell, etc. The shells are rotatably mounted to prevent the innermost nodes from being fret or overused. The contacts (pins) are in their own rotatable shells. The memory addresses are managed like in a virtual memory, which mathematically can also correspond to a memory cube. From time to time the shells are rotated, also if there are failures.

The memory concept presented is also suitable for other hardware components such as hard disks. It enables shorter distances than the conventional one. This is beneficial for indexing database tables, swarm exploration, and threads. The replaceable spherical root unit is designed with multiple redundancies to have a reasonable failure concept. To get to them from the outside, the memory unit can be swung out. A memory tree starts from each root.

Overall, a single memory tree can be (virtually) addressed (so-called *memory duality*). This allows fast sorting methods such as Bitsort (see Theoretical Informatics). For this purpose, the memory cells can possibly be implemented as simpler processor units with their own intelligence. For technical or computational reasons, they can also be designed twice. This concept can be used in a reduced form with PCs. In any case, there is a clear gain in speed.

Theoretical Informatics

The following section presupposes Set Theory and answers for $k, m, n \in {}^{\mathrm{v}}\mathbb{N}$: How can $n = \epsilon^{\sigma} \leq k^{m}$ values be sorted in $\mathcal{O}(1)$ that are coded in the base-k system? How can two matrices $\in {}^{\mathrm{v}}\mathbb{R}^{n \times n}$ be multiplied in $\mathcal{O}(\sigma)$? How can the minimum or maximum framework of a graph with n nodes be calculated in $\mathcal{O}(\sigma)$? What does answer the halting problem? Why is $P = NP$ true?

Let the memory be partially organised in the form of a binomial tree of order $k \in {}^{\mathrm{v}}\mathbb{N}$ with $\mathrm{k}^{\dot{\mathrm{m}}}$ memory locations, where $\mathrm{k}^{\dot{\mathrm{m}}} = 2^{h}$ for $h \in {}^{\mathrm{v}}\mathbb{N}$. Each memory location is able to completely accommodate each value to be sorted and each edge corresponds to a data line. A continuous index is appended to multiple values and forms together with the old value the new value. Each j-th intermediate node corresponds hierarchically with the j-th position of the value on base k to be sorted.

If leaves store (in one step) the values' addresses instead of the values themselves, they can be retrieved, sorted, into the root in a further step. They are maximally pushed into the tree until they separate. If a value passes through an intermediate node, this sets its bit to 1, but to 0 if the intermediate node has no successor to read. The sorting order determines the first value. Saving, checking the successor, masking bits and reading out takes $\mathcal{O}(n)$.

Here and in the following, a memory architecture with autonomous memory units is used that has simple computational capabilities. It can also be simulated by interconnected, distributed computers. Sorting in an additional value increases the complexity by $\mathcal{O}(\sigma)$ since this corresponds to a binary search. It is inversely proportional to the number of tree nodes in the memory. If $d \in {}^{\mathrm{v}}\mathbb{N}$ data lines are used, it decreases by the factor $\tilde{\mathrm{d}}$. If the values occupy g times the number of memory space, it increases by the factor g.

If the memory does not contain a tree of degree k, it increases by the factor ${}_{k}n$, because then every intermediate node must be read to a value. Using the first k^{p} memory locations in another way with $p \in {}^{\mathrm{v}}\mathbb{N}$, requires all intermediate nodes of the p-th stage to be searched through. The complexity increases marginally if the memory is well filled, otherwise, by the worst-case factor $\mathcal{O}(k^{p})$.

Matrices to be multiplied are to be placed in such a way n-square by n-square into a memory cube that every memory unit multiplies two values, and every entry of the product matrix is determined as sum with n summands in $\mathcal{O}(\sigma)$ (vector processor). Multiplying matrices may be also realised by addressed swarming entries. Tensor products can be efficiently represented by storage n-cubes, which remain in three-dimensional space through corresponding data lines.

The optimal sorting algorithm *bit sort* sorts stably with the time complexity $\mathcal{O}(n)$ and requires $\mathcal{O}(n)$ additional storage space. Requirements: Two values are given as bit sequence, can be compared and all their bits processed in $\mathcal{O}(1)$. Further is assumed that the memory is partially realised as a binary tree and that the path from root to leaf can also be traversed in $\mathcal{O}(1)$.

If $p \in {}^{\mathrm{v}}\mathbb{N}$ is the number of parallel steps for a parallel bit sort, the time complexity is $\mathcal{O}(\tilde{\mathrm{p}}n)$. Each insertion into the always sorted tree or output of a value has the time complexity $\mathcal{O}(1)$. The memory is provided with the following intelligence: each tree node stores the value reaching it independently and can pass it on to the next tree node if it has already been visited, or, otherwise, can return its address to the central unit. The root does not correspond to any bit value.

Starting with the most significant bit of a value, each of its consecutively processed bits determines the next node within the binary tree. If all bits have been processed or if there is a difference from the compared bit and the associated node has not yet been visited, the non-tree address of the remaining bit string of the value is stored in the following node if any, or the number of values for this node, given that equal values need not be ordered.

The time complexity of all methods which sort values using a bit length $b = qa \in {}^{\mathrm{v}}\mathbb{N}$ alongside an address length $a \in {}^{\mathrm{v}}\mathbb{N}$ increases from $\mathcal{O}(1)$ to $\mathcal{O}(q)$ for each value. If r is the average value of q, at least one $\mathcal{O}(n)$ must be replaced by $\mathcal{O}(rn)$ in the time complexity of the sorting procedure. If $s \in {}^{\mathrm{v}}\mathbb{N}$ is the average number of different (!) values that end up in a bucket after sorting to bit length a and whose bit length is greater than a, then the time complexity of the procedure is $\mathcal{O}((_{\epsilon}s + r)n)$.

In the process, the mentioned values of a bucket are stored in one AVL tree per bucket or in a B-tree, if they are stored in a memory which is relatively slow compared to the main memory. There, only the significant part value of the address length of a value is processed. If is dispensed with completely sorted values after every processing of a value, the complete sort is temporally reduced to $\mathcal{O}(rn)$ by using two main memory areas.

The second main memory area is cleared after each pass so as to be available for the next pass until a bucket has been processed. In the first main memory area, the addresses are always just written back in the current sorting order before further sorting the values with the same part value just processed. Each value must be stored contiguously in memory, so that each part value can be accessed quickly.

The use of the $\mathcal{O}(rn)$ method for the master data and the $\mathcal{O}((_{\epsilon}s + r)n)$ method for the update data both combine well. In particular, table indexes can be created quickly. For large tables, it makes a significant difference whether an index is created in $\mathcal{O}(rn)$ instead of in $\mathcal{O}(r\sigma n)$, and whether a value is then on average searched for in $\mathcal{O}(_{\epsilon}s + r)$ instead of in $\mathcal{O}(\sigma + r)$ or worse. Sorting by mixing sorted lists is also possible in $\mathcal{O}(rn)$.

Discussion of bit sort: It is optimal under the given conditions for parallel programming, and well suited both for the rapid ongoing building of table indexes in databases and for search. Its hardware implementation is rather unusual.

Let $n \in {}^{\nu}\mathbb{N}$ be the node number of a finite simple connected graph. (If it is not simple, only the smallest parallel edge and no loops are allowed by sorting out in $\mathcal{O}(n)$. If it is not connected, all isolated nodes are removed in $\mathcal{O}(n)$.) If negative weights are excluded, Dijkstra's algorithm needs $\mathcal{O}(n)$ steps for every minimum path in the graph, since sorting as well as parallel adding and comparing is possible in $\mathcal{O}(1)$.

Its minimum or maximum framework can be calculated in $\mathcal{O}(\sigma n)$ or even in $\mathcal{O}(\sigma)$. The edges of a node can be sorted in parallel in $\mathcal{O}(n)$ or $\mathcal{O}(1)$. Let all edges be coloured white at the beginning. Each node appends the smaller and then the larger number of participating nodes of the edge with the smallest weight of all the white-coloured edges that emanate from it to this bit by bit, as a value. It stores the smallest value that reaches it. This is possible in $\mathcal{O}(n)$ or $\mathcal{O}(1)$.

Blackening of the edges between every two nodes of the same minimal framework requires at most the same time. Each node then has a nearest neighbouring node. Continuing with the white coloured edges in $\mathcal{O}(\sigma)$ steps, the smallest value of a minimal framework can be determined in $\mathcal{O}(n)$ or $\mathcal{O}(1)$ until the entire minimal framework is formed in $\mathcal{O}(\sigma n)$ or $\mathcal{O}(\sigma)$. For a maximum framework, we subtract all edge weights from a sufficiently large constant.

Halting theorem: To reach a fix r_m of the number $0.r_1r_2...r_m$, a programme halts for arbitrary (pseudo-) random numbers $r_j \in {}^{\nu}\mathbb{N}$ with $k \in {}^{\nu}\mathbb{N}^*$ positions and base $p := 2^k$ in $m \in \mathbb{N}^*$ steps giving probability $\tilde{p}$.

Proof: This is an optimal result of probability calculation (cf. [973], S. 10 - 40).□

Remark: Chance is undecidable. This means that there are programmes halting with any probability, and Gödel's incompleteness theorems follow. The Gödel sentence "I am not provable." is sophistical in that it has the bogus argument petitio principii ex negativo. In order to solve the halting problem, a self-reference is senseless, which commands and prohibits halting at the same time. The self-reference makes the diagonal argument for the holding problem undue.

Theorem: If $\mathcal{L}$ is the set of all (type 0) languages $\mathcal{L}_k$ of finite words with maximum length ν for mid-finite $|\mathcal{L}| = 2^{(\nu^{\nu+1}-1)/\acute{\nu}}$ (using the geometric series) and $k \in \{1, ..., |\mathcal{L}|\}$, then $P = NP = \mathcal{L}$.

Proof: Since every deterministic Turing machine can also be considered non-deterministic (cf. [972], p. 162, 318 and 361 f.), $P \subseteq NP$ is true. It contains P all linear languages $\mathcal{L}_k$, and NP only those ones.□

Mathematics

The following results in the branches set theory, topology, nonstandard analysis, number theory, linear programming, scientific computing, Euclidean geometry and theoretical informatics are brilliant achievements! Known statements and elementary concepts such as axiom, field, etc., are as given in the relevant literature or on Wikipedia. Hence, only deviating or clarifying definitions will follow.

Unlike conventional usage in which brackets denote a more detailed explanation, bracketed parts of a statement can either be included or excluded: the statement is valid in both cases. The symbol $\square$ finishes a proof whereas $\triangle$ terminates a definition. The world's finiteness creates certain difficulties to handle the infinite. The largest exponent of a polynomial's argument is its degree. Minimal polynomials may be specified by integer (instead of rational) coefficients as throughout in the following text.

Preliminary considerations: Referencing to philosophical terms is justified since mathematics cannot be explained in terms of metalinguistics alone and abstract words provide the most general statements according to the principle of scientificity. Here the word successive contains the physical idea that the time span between consecutive points in time is postulated, measurable or perceptible in some form. In this world, it corresponds to the Planck time of about 10^{-43} seconds.

The reproach of lacking general validity by including physical facts must be countered that the infinite, as a dual concept to the finite, can only be explained by a non-terminable process or a spatial equivalent being counterparts of the instant or the same place. Due to the structurally similar quantities of space and time combined in space-time, the statements below concerning space apply to time, too.

In practice (of computer science), sufficiently small formal systems often help. The abrupt transition from finite to infinite numbers, which is difficult to convey, requires mid-finite ones. The existence of actual or only potentially infinite sets remains open, since the transcendence of the infinite fails to provide a proof. From the almost "wasteful" size or expansion of the universe, it can be concluded that there is an availability that has no recognisable limits.

However, such inferences are weaker than abduction. If a finite line is broken down into an infinite number of parts, the infinite is finitely limited. If, in addition, the number of parts of both infinities is the same, there is mathematically an isomorphism: The enlargement of the infinitely small parts of the finite segment to finite ones results in infinity in the conventional sense in relation to the whole. That easily deduces a bijection in the mathematical sense.

Rational numbers are real, put a minimum polynomial of degree 1 to 0 and some have periodic decimal places. If the degree is ≥ 2, these are purely algebraic numbers with infinite numerators and denominators. Finite real fractions, together with the infinite or purely complex fractions, already form all real or complex numbers. Real continued fractions that do not terminate as finite fractions are algebraic (conventionally transcendent!) since they have an infinite denominator.

Algebra shows that sum, difference, product, and quotient of two conventional algebraic numbers of natural degree m or n are algebraic of degree at most mn, and that the $1/m$-th power of a conventional algebraic number of degree n is also algebraic of degree at most mn. If the algebraicity of a number is to be investigated and the remainder is given by the limit of a zero sequence (a_n), omitting the sequence values for large n is not permitted. They are crucial.

Since all real numbers are approximately finite numbers, they can be calculated in real-time. An infinite real number can consist of a finite continued fraction with an infinitely large last denominator. If it were equated to a conventionally rational number by removing the last partial fraction, it would also be the solution to a linear equation with (infinite) whole coefficients.

Associative, commutative, and distributive laws allow to arbitrarily rearrange sums if care is taken to calculate them correctly (using Landau symbols). The (finite) definition offers significant advantages of handling and is traditional. It is shown that the sets of natural, integer, algebraic, real and complex numbers are not closed. The minimality theorem explains the choice of 2 as basis (also for digital computers that are working most often within a binary system).

The correct treatment of bijections yields the statement that, concerning the number, there are infinite many sets between the set of conventionally natural numbers and the one of conventionally real numbers. The definition of real numbers via Dedekind cuts is thus just as redundant as its definition via equivalence classes of real Cauchy sequences. Thus, the continuum hypothesis gets a new answer. The whole set theory is of course more extensive since only essential (new) ideas are presented here.

A disk without its boundary conventionally represents an open set, because then each point of it has a conventional neighbourhood that lies completely in this set. If the points are considered on a half-line, starting at the centre of the disk, there must be always a real neighbourhood for each point on this half-line towards the boundary.

This idea so far negates, however, "the end of the flagpole" whereby every such half-line must contain a point in the interior of that disk without any conventional neighbourhood there. Hence, the term openness for sets is inept (cf. [819], p. 36). If the unit disk is considered around the origin of ordinates, so the last point of the half-line [0, 1[, dually represented, is the point $0.\bar{1}_2$ and the next point is the boundary point 1. There is no other point between these two points.

For this reason, the disk without boundary is also closed, since the considered end points of the half-line just form the closure as boundary. Because the neighbourhoods do not exist on their boundary, the term closure is meaningless for sets in Euclidean space. At the same time, every open set is also closed there. This absurdity is unsettling when infinitesimal quantities are considered in a differentiated manner, thus in particular the numbers 1 (rational!) and $0.\bar{1}_2$ (algebraic!) are not equated.

The absurd shows itself also by an infinite intersection of open sets such as by all open concentric disks forming a closed set, more precisely: the common centre of the disk. An infinite union of closed sets can build an open set as an open disk does, as a union of all its points as closed sets. A 0-dimensional set (point) is therefore open, because every neighbourhood also consists of one point.

Hence, the empty set $\emptyset$ is also closed, and consequently the complete Euclidean space is closed, what can easily be generalised to higher dimensions for spheres. This special case makes also the general one absurd and meaningless: considering open or closed sets is not suitable for metric and topological spaces. Particularly, the definition of a conventional topological space appears oddly content-free and arbitrary.

Permitting infinitesimal radiuses whose reciprocal are infinite make the terms of the inner and outer point meaningful, however. The conventional irrationality proof of $\sqrt{2}$ shows some dialectics and $\mathbb{R}$ contains the multiplicative inverses usually only approximately. The subject is also about conventionally unmeasurable, mid-finite and infinite sets as well as discontinuous functions. Every probabilistic statement is only valid when all its relevant possibilities were verified.

When integrating identical paths in opposite positive and negative directions, the counter-directional rule for integrals is adopted, stating that when following the path in the negative direction, the same (!) function value of both possible ones must be chosen if the function is too discontinuous, implying that the integral sums to 0 to prevent a significantly different value. Stokes' theorem may be proven generally (cf. [764], p. 625 f.). Function values may skip values, which are not measured then.

Conventional differentiation and integration lose the ability to distinguish between rationality and pure algebraicity in the conventional process of taking limits. This is e.g. problematic when setting about determining the roots of a polynomial exactly. Therefore, the conventional analysis cannot be preserved in its existing form and requires practicable alternatives. Period 2π must define sine and cosine since their power series only converge for finite arguments.

The exact volume integral is the easiest to handle compared to conventional ones. Its improper form arises analogously. In some cases, suitable Landau notation may be useful. Combining function values to finitely many continuous functions, integrals may be calculated even for discontinuous functions. Here, one of both Euler-Maclaurin formulas may be used. Note that the continuity of neither integral nor derivative is assumed. An appropriate definition makes this possible.

If the result of differentiation lies outside of the domain D, the closest number within the domain D should replace it. If this is not uniquely determined, the result can either be given as the set of all such numbers, or the preferred result may be selected (e.g. according to a uniform rule). The exact integral is more general

than Riemann, Lebesgue(-Stieltjes) integral and other types of integral. The latter exist only in conventionally measurable sets. Extrapolations may detect bends.

Using $1/\infty$ instead of 0 avoids a division by 0 and any vague notions of limits but requires considering carefully where this replacement makes sense to invoke no contradiction by switching the symbols. This also allows to define integrals and differentials for each operation on real and complex numbers in such a way that every function is at least directionally integrable and differentiable wherever the function values are defined.

The definition of the exact integral via a rectangular rule may require error estimations (cf. [564], [945] and [742]). Actual integration as the inverse operation to differentiation only makes sense for continuous functions if it should go beyond simple summation. The term predecessor, which is dual to successor, is usually not specifically mentioned again, but must be kept in mind. Only three examples illustrate the superiority of nonstandard analysis and the strength of using infinitesimal and infinite values.

The Riemann series theorem is false, since the commutative law cannot be avoided, even at infinity. It is unjustifiable arbitrariness if single summands are shifted to negligible infinity without being further considered. Coefficients of Taylor series may be computed very easily (for complex numbers) by using discrete Fourier transform (DFT). DFT forms derived from this are suitable for efficient number representation and calculation.

The Taylor series of the integrated logarithm, as a geometric series, allows a simple implementation of the multiplicative inverse. Functions can be determined as a vector or product of the Fourier matrix, which is a Vandermonde matrix of roots of unity, by a vector of fixed function values at the so-called crown points. The latter are arranged in a circle around a crown centre, which should be chosen effectively and can contribute to good convergence.

The product mentioned can be stored before the calculation with a freely selectable precision, which for computer calculations is best the reciprocal of a power of two. Since it can be calculated quickly and Taylor series can be realised using the Horner scheme, this method (especially as an FFT version) is very efficient. Linear algebra also achieves this efficiency when solving most differential equations numerically using DFT forms.

In partial differential equations, the variables of dimensions other than one are fixed. Here tensors replace matrices. Absolute values are considered by using the signum function. This book is based on ISO 80000-2:2019 (quantities and units - mathematics). The conventional notation of sums (Σ), products (Π), differentials (d and ∂), integrals ($\int$) and roots ($\sqrt{}$) is prevented: It is too sweeping and historically less intuitive.

Simplex and numerically easy intex methods, which were developed in 35 years and answer (e.g. with the help of Gomory cuts) Hilbert's tenth problem positively, can solve a linear programme mostly in cubic and quadratic time respectively. The simplex method allows easy and precise computing for smaller initial problems with (approximated) real fractions.

Since individual n-ness belongs to every natural number n that cannot be derived from its predecessors or successors, there is no complete system of axioms in mathematics, because something irreducible new emerges with each new number. By confining, however, to selected aspects, a finite system of axioms can be specified for a finite number of entities. Each level of infinity refuses completeness even more.

Mathematics is not value-free. Theories are based on presuppositions. In mathematics, they are willingly expressed by axioms demonstrated to be true or false. Thus, all theories are incomplete and, as the case may be, beyond that, contradictory. Instead of explicit axioms, implicit definitions are more suitable in that the existence of the specified is tacitly presupposed with all emerging difficulties, until refutation.

The Euclidean geometry gives three definitions that challenge several axioms, using results of set theory. The question of a fair distribution of persons deals less about the theoretical content than about practical application. Here, a spreadsheet analysis is used. The set theory defines some new sets and counts generally their elements, especially the algebraic numbers.

The nonstandard analysis redefines integration, differentiation, continuity, convergence and limit value for finite and infinite, and maybe conventionally non-measurable sets, and for discontinuous functions. Sometimes, it considers their homogeneity as well. It gives examples to obtain better, more precise, more elegant

or simply correct mathematical statements, also for known theorems of analysis. It solves the measure problem.

The advice is the result of my mathematical experience and discusses appropriate ways and procedures for solving mathematical problems. The topology redefines the (simple) connectedness of sets and states the real and complex numbers as spaces only with Fréchet topology resp. without Hausdorff property.

In number theory, necessary and sufficient criteria for transcendental numbers are stated, giving some new examples, too. The limit, coefficient and approximation theorem and the largest prime criterion are proved. Roth's theorem is disproved. A conjecture by Littlewood is refuted and conventionally proven. The conjecture by Alaoglu and Erdős is proved, and the generalised Riemann hypothesis is disproved.

In the computation of time, it is shown how the octal system can be used practically worldwide and what the advantages are here. Besides introducing a new calendar and a new calculation of clock time, the octal system is applied also to the SI-units metre (m) and second (s). It is reasoned with practical examples what makes the octal one is advantageous by completely replacing the decimal system.

Beauty and elegance in mathematics can be ensured by adequately thinking through what is to be presented, and without being stingy, reducing it to the clear essence, which justifies both and is a hallmark of the true. Unfortunately, there is a lot of ugly length in the mathematical world. It can only be hoped that this book can provide a lot of pleasure with the nonstandard mathematics to give insight into the real good and beautiful. Who likes it, may realise both of it!

Euclidean Geometry

The following section presupposes Set Theory with the Euclidean norm $||\cdot||$.

Definition: Let R a Euclidean space as a subspace of $\mathbb{R}^n$ with $n \in {}^{\omega}\mathbb{N}_{\geq 3}$ (see Set Theory) Two distinct *points* as elements of R are called *pair (of points)*. A one-dimensional point set is called a *line* if each point has at least one and at most two gaplessly neighbouring points. A one-dimensional subspace of R is called *straight line,* a two-dimensional one is said to be a *plane*. The *distance* is given by the Euclidean norm $||\cdot||$ and results summed up in the Euclidean *arc length* A.△

Definition: A *line segment* is a connected subset of a straight line, whose *starting point* and *end point* determine it precisely and have to only one neighbour finite distance. Two line segments are said to *intersect* if they have one interior point in common. Two line segments are said to be *parallel* if one line may be obtained from the other by means of a translation. All points of a plane with the same distance (called *radius*) to another point constitute a *circle*, which forms with its interior a *disk*.△

Result: By defining short straight lines, many counterexamples can be given based on the above to Pasch's axiom, the axiom of line completeness, as well as various other axioms and their equivalents. The subspace concept can prove Hilbert's incidence axioms I.6 and I.8. If a straight line uniquely defines a parallel straight line running through a given point by their shortest distance, the parallel postulate is dispensable in Euclidean Geometry. The axioms of order and congruence are redundant.

Contrary to Hilbert's incidence axiom I.4, three distinct points are not enough to uniquely determine a plane since there are different infinities. Hilbert's incidence axiom I.7 is false (cf. Hilbert, David: *Grundlagen der Geometrie*; 2. Aufl.; 1903; Teubner; Leipzig, p. 2 - 17), since planes are yet limited in infinity and can precisely intersect in one point. All (three ancient) exactly unsolvable problems may be solved with arbitrary precision by the fundamental theorem of set theory via the intercept theorem and Farey sequences ([455], p. 62 f.).

Rating two straight lines only as parallel when they lie in the same plane and do not intersect, the parallel postulate does not hold: The reciprocal of the distance between the straight line and the given point may be greater than infinity or smaller than $|{}^{\omega}\mathbb{N}|$, and then infinitely many distinct straight lines can be found that pass through the given point without intersecting the original straight line. Formulating the axiom of completeness vaguely, Hilbert renders it finally unapt in future.

The Archimedean axiom must be extended to the case where a segment is marked off an infinite natural number of times without exceeding the starting point or end point of a straight line. It must be in the finite case replaced by the Archimedean theorem (see Set Theory). Pasch's axiom is also unnecessary, since every straight line must be fully contained in the interior of some triangle due to its maximum length, and hence so must its boundary, provided that one of its points is in this interior.

Normal theorem: For $n \in {}^{\omega}\mathbb{N}_{\geq 2}$, every convex set S from ${}^{\omega}\mathbb{R}^n$ contains a point lying on $\hat{\mathrm{n}}$ distinct normals through ∂S (also through a vertex), since this is possible every 90° in the plane (cf. [931], p. 14 f.).□

Toeplitz' conjecture: Every Jordan curve admits an inscribed square.

Counterexamples: The right-angled triangle with two sides of length ι and the obtuse triangle where a vertex of at most one inscribed square is infinitesimally moved within the limits.□

Theorem: Infinitesimally juxtaposing equichordal points within the limits leads to a Jordan domain with more than one equichordal point (cf. [931], p. 9 f.).□

Fickett's theorem: Any relative position of two overlapping congruent rectangular n-prisms P and Q (see [931], p. 25) with $n \in {}^{\omega}\mathbb{N}_{\geq 2}$ and $\dot{\mathrm{m}} := \hat{\mathrm{n}}$ implies for the exact standard measure μ, which is A for $n = 2$:

$$\tilde{\mathrm{m}} < r := \mu(\partial P \cap Q)/\mu(\partial Q \cap P) < m.$$

Proof: The underlying extremal problem has its maximum for rectangles with the side lengths s and $\mathrm{s} + \hat{\iota}$. Putting $q := 3 - \hat{\iota}\tilde{\mathrm{s}}$ implies $\min r = \tilde{\mathrm{q}} \leq r \leq \max r = q$. The proof for $n > 2$ works analogously.□

Equitable Distributing

In life, there is often distribution unfairness. Although the following is not new, equitable distributing is to be demonstrated by examples that occur daily at school:

1. How do we distribute work-results equitably?
2. How do we channel groups equitably through the cafeteria?
3. How do we form equitably groups in physical instruction?

To 1.: We first put down the work results alphabetically, arranged according to the names of the present pupils (in trays, on the table or the like). The names are or will be put down alphabetically in a spread-sheet analysis (except the missing pupils). Behind each name, a random number is produced. The names are arranged according to the random number. We call the pupils according to this order (possibly in groups) so that they can receive their work results.

To 2.: At the beginning, we form groups with members that would like to dine together or are to do this. These groups or their names are arranged as described under 1. Then for each group (the available output time) * group size / diners is computed. The second group goes into the cafeteria if the time determined for the first group is up. The third group follows if additionally the time for the second group has elapsed and so on. If necessary, the points in time have to be computed rounded. If singular groups have time only in certain intervals, they are distributed first on the time intervals. Then the procedure continues with the remainder time for the groups left.

To 3.: At the beginning, groups of same capacity are formed and the individual names are written in a firm sequence one below the other and serially numbered due to an assigned group number. If group members are missing, we filter them first by noting e.g. the entry "missing" behind their name and selecting the empty entries afterwards.

The group size should be a multiple of the number of groups that can be formed. The sequence of the names should be firm, for reasons of the probability. Changes of the capacity are considered in changed groups. Behind each name, the numbering is divided by the number of groups. If the numbering does not begin with 0, a number must be subtracted before accordingly. The decimal places of the received values are truncated. This can be attained also by a rounding of the values reduced by 1/2. In a further column, now random numbers are registered.

Thereupon the three columns are arranged according to the values determined first, then additionally according to the random numbers. In a fourth column, beginning from above, the numbers from 1 to number of groups are registered. After the last value, it is again continued with 1 up to the number of groups and so on. Now the numbers are sorted again according to names. Finally, we can announce the group assignments in alphabetical order.

If the number of pupils to be divided is not a multiple of the number of groups, we should form proxies from the same group for each name. The namebearer who have the smallest time absent by sitting out are intended to sit out. They represent their deputies (e.g., if there are several runs in an instruction unit). The time absent is credited accordingly.

A simple procedure with a dice (instead of a computer) functions as follows: The pupils are set up ordered according to their (self-determined or over-directed) capacity on a line. The one with the highest capacity is assigned to one of the groups by throwing the dice. The next one is determined under the remaining groups by the dice and so on. If each group has a member, it is continued as with the one with the highest capacity now with the one having the henceforth highest capacity.

Without considerable aids, we can form groups also as follows: We request all pupils to move at random each time in a circle after their capacity. We prevent couple formation by the request "A and B apart to the circle line!". Everyone remains at her or his place after the command "stop!". We now request one of the pupils to leave her or his place and to move on an approximate circle line around the remaining pupils or around a group each time.

After a further command "stop!", we divide the pupils by imaginary straight lines into groups. First, the intersection is built with the circle line of the group the straight line forms through the break point of the

one who stands on the circle line and the respective centre of the circle. The groups are defined by parallel straight lines that go parallel to the straight line through the respective intersection and the respective centre of the circle.

These procedures are more equitable than the judging selection by individual pupils. Three capacities are sufficient according to experience.

Linear Programming

In the following section, Set Theory, Topology and Nonstandard Analysis are presupposed.

Diameter theorem for polytopes and polyhedra: Every diameter of an n-dimensional polytope or polyhedron defined by m constraints for $m, n \in {}^{\omega}\mathbb{N}_{\geq 2}$ is at most $2(m + n - 3)$.

Proof: At most ḿ hyperplanes can be assembled into an incomplete cycle of dimension 2 and there are at most $n - 2$ alternatives sidewards in all remaining dimensions. Overcoming every minimal distance requires at most two edges and yields the factor 2.□

Definition: Let $\vartheta := \omega_{\epsilon}\omega$ and $\|\cdot\|_1$ the 1-norm. If $x \in X \subseteq {}^{\omega}\mathbb{R}^{\omega}$ passes to the next step, it is followed by * where $\Delta x := x^* - x$. The *unit vector* of $x \in X^*$ is ${}_1x := x/\|x\|$. Let $x, y \in X$ in $(x, y, \ldots)^T$ be row vectors. If a method requires computation time in seconds and memory in bits of $\mathcal{O}(\omega^{\mathcal{O}(1)})$, it is *polynomial*. If one of both quantities is $\mathcal{O}(\epsilon^{|\mathcal{O}(\omega)|})$, the method is *exponential*. Let the *eigenproduct* (yet: determinant) of a square matrix be the product of its eigenvalues.△

Theorem: The simplex method from [777] may solve an (LP) almost always in $\mathcal{O}(\omega^3\vartheta)$.

Proof and algorithm: Let LP $\max\{c^Tx : c \in {}^{\omega}\mathbb{R}^n, x \in P\}$ have the feasible domain $P := \{x \in {}^{\omega}\mathbb{R}^n : Ax \leq b \in {}^{\omega}\mathbb{R}^m, A \in {}^{\omega}\mathbb{R}^{m\times n}, m, n \in {}^{\omega}\mathbb{N}^*\}$. Its dual gives $x^* \in {}^{\omega}R^m_{\geq 0}$. A (big) $x^+ \in {}^{\omega}R_{\geq 0}$ achieves $x^* \in {}^{\omega}R^n_{\geq 0}$ for $x := x^* - x^+1_n$. Solving $\max\{-h \in {}^{\omega}R_{\leq 0} : Ax - b \leq h1_m\}$ yields $x \in P_{\geq 0}$ if $b \geq 0$ does not hold. Let $|\min\{b_1, \ldots, b_m\}|$ be the initial value for h and 0 its target value. Start with $x_0 := 0$. Only one pivot step causes that $b^* \geq 0$. Let $i, j, k \in {}^{\omega}\mathbb{N}^*$ and let a_i^T the i-th row vector of A. If $c_j \leq 0$ for all j, the LP is solved.

If for some $c_j > 0$ also $a_{ij} \leq 0$ for all i, the LP is positively unbounded. If for some $c_j = 0$ also $a_{ij} \leq 0$ for all i, drop c_j and $A_{.j}$ as well as b_i and a_i, but only when $a_{ij} < 0$ holds. The inequality $a_{ij}x_j \geq 0 > b_i$ for all j has no solution, too. Dividing all $a_i^Tx \leq b_i$ by $\|a_i\|$ and all c_j and a_{ij} by the minimum of $|a_{ij}|$ such that $a_{ij} \neq 0$ for each j maybe repeated and reversed later. Constraints with $a_i \leq 0$ may always be removed.

For each $c_j > 0$ and non-base variable x_j, select $b_k/a_{kj} := \min\{b_i/a_{ij} : a_{ij} > 0\}$ to obtain a next $x^* \in P_{\geq 0}$ where $x_j^* = x_j + b_k/a_{kj}$. To select the steepest edge, pick the pivot a_{kj} corresponding to x_j that maximises $c^T{}_1\Delta x$ or $c_j^2/\grave{v}_j^2$ for $v_j := \|A_{.j}\|$ in the k-th constraint. Multiple maxima allow to use the rule of best pivot value $\max_{k,j} c_j\, b_k/a_{kj}$ or (slower) the smallest angle $\min {}_{(1)}11_n^T\, {}_1c^*$.

If $c^Tx^* = c^Tx$ is repeated, relax all constraints with $b_i = 0$ by $d \in {}^{\omega}R_{>0}$. For $b_i := \|a_i\|$, drop d in the tableau. If another multiple vertex is encountered for once, simply add $\|a_i\|$ to every b_i. Leaving it may require to solve an LP with $c > 0$ and $b = 0$. Then the relaxation (perturbation) is reverted. This task can also finally arise, if further solutions of the LPs are to be determined for at least two $c_j = 0$.

This avoids having to compute several minimal relaxations that differ strongly as the lexicographical method does. The rectangle rule simply allows to compute c_j^*, a_{ij}^* and b_i^* (cf. [775], p. 63). Bland's rule is not optimal, too. Linear programming cannot prevent a maybe exponential runtime: Klee-Minty or Jeroslow polytopes e.g. have (remote) vertices, which numerically make not much of a difference. The result follows in accordance with the state of research.□

Bit-square theorem (bit² theorem for short): Repetitions allow more than $\mathcal{O}(n^2)$ multiplications and additions to be reduced to $\mathcal{O}(n^2)$ for an algorithm of numbers of bit length $m \leq n$ with $m, n \in {}^{\omega}\mathbb{N}^*$.□

Examples: The matrix product $(c_{ij}) := \left(+_{k=1}^{n} a_{ik}b_{kj}\right)$ of (a_{ij}), $(b_{ij}) \in {}^{\omega}\mathbb{C}^{n\times n}$ over products $+_{k=1}^{n} b_{jk}4^{km}$ with a_{ij} and $i, j \in {}^{\omega}N^*_{\leq n}$, determining an n-dimensional Taylor series (see Nonstandard Analysis), the Gauss-Jordan algorithm, the simplex and the Gram-Schmidt orthogonalisation method.

Remark: If m is too large, it will be appropriately decreased computationally. Coding quaternions enables concerted matrix multiplication in the cloud by comparing simultaneously.

Theorem: Intex methods solve almost every solvable LP with *inter*-/*ex*trapolations in $\mathcal{O}(\vartheta^3)$.

Proof and algorithm: Let $z := m + n$ and $d \in [0, 1]$ the density of A. First, normalise and scale $b^{T}y - c^{T}x \leq 0$, $Ax \leq b$ as well as $A^{T}y \geq c$. Let $P_r := \{(x, y)^{T} : x \in {}^{\omega}\mathrm{R}^{n}_{\geq 0}, y \in {}^{\omega}\mathrm{R}^{m}_{\geq 0}, b^{T}y - c^{T}x \leq r \in [0, \rho], Ax - b \leq r1_m, c - A^{T}y \leq r1_n\}$ have the radius $\rho := s|\min\{b_1, \ldots, b_m, -c_1, \ldots, -c_n\}|$ and the scaling factor $s \in]1, 2]$.

It follows $01_z \in \partial\mathrm{P}_{\rho}$. By the strong duality theorem ([775], p. 60 – 65), LP $\min\{r \in [0, \rho] : (x, y)^{T} \in P_r\}$ solves LPs $\max\{c^{T}x : c \in {}^{\omega}\mathbb{R}^{n}, x \in \mathrm{P}_{\geq 0}\}$ and $\min\{b^{T}y : y \in {}^{\omega}\mathrm{R}^{m}_{\geq 0}, A^{T}y \geq c\}$. Its solution is the geometric centre g of the polytope P_0. For $\mathrm{p}^{*}_{\mathrm{k}} := \min \check{\mathrm{p}}_{\mathrm{k}} + \max \check{\mathrm{p}}_{\mathrm{k}}$ and $k = 1, \ldots, \grave{z}$ approximate g by $p_0 := (x_0, y_0, r_0)^{T}$ until $\|\Delta \mathrm{p}\|_1$ is sufficiently small. The solution $t^{o}(x^{o}, y^{o}, r^{o})^{T}$ of the two-dimensional LP $\min\{r \in [0, \rho]: t \in {}^{\omega}\mathrm{R}_{>0}, t(x_0, y_0)^{T} \in P_r\}$ approximates g better and achieves $r \leq \check{\rho}$.

Repeat this for $t^{o}(x^{o}, y^{o})^{T}$ and so forth until $g \in P_0$ is computed in $\mathcal{O}(2\rho^{2}dmn)$ if it exists. Solving all two-dimensional LPs $\min_k r_k$ by bisection methods for $r_k \in {}^{\omega}\mathrm{R}_{\geq 0}$ and $k = 1, \ldots, z$ in $\mathcal{O}(\vartheta^{2})$ each time determines $q \in {}^{\omega}\mathbb{R}^{k}$ where $q_k := \Delta p_k\, \Delta r_k / r$ and $r := \min_k \Delta r_k$. Let simplified $|\Delta p_1| = \ldots = |\Delta p_z|$. Here $\min \mathrm{r}_{\grave{z}}$ for $p^{*} := p + wq$ and $w \in {}^{\omega}\mathrm{R}_{\geq 0}$ may be solved, too. If $\min_k \Delta r_k r = 0$ follows, stop computing, otherwise repeat until $\min r = 0$ or $\min r > 0$ is sure.□

Remark: Details are only published when no misuse for non-transparent or bad decisions must be feared. Simplex method and face algorithm ([916], p. 580 f.) may solve the LP faster for small m and n. Intex methods are faster than all known (worst-case) LP-algorithms in $\mathcal{O}(\omega^{37/18}\vartheta)$. Rounding errors provoked in b and c make any solution unique.

Remark: If necessary, the same small modulus relaxes constraints temporarily. The runtime for $m, n \in {}^{\nu}\mathbb{N}^{*}$ is $\mathcal{O}(dmn)$. Optimising it for distributed computing in ${}^{\nu}\mathbb{R}^{\nu}$, reduces it to $\mathcal{O}(1)$. Integer solutions v_j satisfying $\acute{v}_0 \leq \lfloor v_j \rfloor - v_j$ prove them successful for (mixed) integer problems and (non-) convex (Pareto) optimisation (according to nature as in [1010]).

Remark: Intex methods make use of the fact that the calculated geometric centre and the centre of the sphere coincide in P_0. The former is secured by rotating the coordinate axes (in pairs) like c with the aid of a simple rotation matrix in a comparable runtime. Numbers of length $\mathcal{O}(\omega)$ can only be processed in $\mathcal{O}(\vartheta)$ as is generally known. The following results can also be easily transferred to complex numbers:

Conclusion: LP $\max\{\|\mathrm{x}\text{-}\mathrm{x}^{o}\|_1 : c^{T}x = c^{T}\mathrm{x}^{o}, Ax \leq b, x - \mathrm{x}^{o} \in [-1, 1]^{n}, x \in {}^{\omega}\mathrm{R}^{n}_{\geq 0}\}$ can determine for the first solution x^{o} any second one in $\mathcal{O}(\vartheta^{3})$ where y^{o} may be treated analogously.□

Conclusion: LP $\max\{\nu|\lambda_j| + \|\mathrm{x}_{\mathrm{j}}\|_1 : Ax_j = \lambda x_j, |\lambda_j| \in [0, r_j], x_j \in [-1, 1]^{n*}\}$ can determine for every $j = 1, \ldots, n$ the eigenvalue $\lambda_j \in {}^{\omega}\mathbb{R}$ and the eigenvector $x_j \in {}^{\omega}\mathbb{R}^{n}$ of matrix $A \in {}^{\omega}\mathbb{R}^{n\times n}$ in $\mathcal{O}(\vartheta^{3})$.□

Conclusion: LPs $\max\{x_j : Ax = 0\}$ yield all solutions to $Ax = b$ in $\mathcal{O}(\vartheta^{3})$. Matrix A is regular if and only if LP $\max\{\|\mathrm{x}\|_1 : Ax = 0\} = 0$.□

Conclusion: Let $\alpha := \mathrm{A}^{-1}_{\cdot\mathrm{j}}$ for matrix $A^{-1} \in {}^{\omega}\mathbb{R}^{n\times n}$ and let δ_{ij} the Kronecker delta where $j = 1, \ldots, n$. A regular A has an eigenproduct $\neq 0$ and allows every $A\alpha_j = (\delta_{1j}, \ldots, \delta_{nj})^{T}$ to be solved in $\mathcal{O}(\vartheta^{3})$.□

Conclusion: The polynomial $\mathrm{c}_{\acute{n}}\mathrm{x}^{\acute{n}} + \ldots + \mathrm{c}_1 x + \mathrm{c}_0$ can be determined in $\mathcal{O}(\vartheta^{3})$ by the measurement $(x_j, y_j) \in {}^{\omega}\mathbb{R}^{2}$ and residues $r_j \in {}^{\omega}\mathbb{R}$ for $j = 1, \ldots, m$ and $(c, x)^{T} \in {}^{\omega}\mathrm{R}^{\grave{n}}$ from $y = r + Xc$ and $X^{T}r = 0$ where $X = \left(\mathrm{x}^{\mathrm{k}}_{\mathrm{j}}\right)$ for $k = 0, \ldots, \acute{n}$ (also under additional constraints, cf. [893], p. 196 ff.).□

Conclusion: The DFT (see Scientific Computing) can determine the Padé approximant $\mathrm{a}(\mathrm{x})/\mathrm{b}(\mathrm{x}) = +^{\mathrm{n}}_{\mathrm{m}=0}\, \mathrm{a}_{\mathrm{m}}\mathrm{x}^{\mathrm{m}} / +^{\mathrm{n}}_{\mathrm{m}=0}\, \mathrm{b}_{\mathrm{m}}\mathrm{x}^{\mathrm{m}}$ of $\mathrm{f}(\mathrm{x})$ for $x \in {}^{\omega}\mathbb{R}$ in $\mathcal{O}(\vartheta^{3})$.□

Corollary: Intex methods and two-dimensional bisection or Newton's methods may solve most solvable convex programmes $\min\{f_1(x) : x \in {}^{\omega}\mathbb{R}^{n}, (f_2(x), \ldots, f_m(x))^{T} \leq 0\}$ where all $f_k \in {}^{\omega}\mathbb{R}$ are convex functions for $k = 1, \ldots, m$ in polynomial runtime, if the number of operands x_j of all f_k is $\leq \omega^{\nu-3}$ and if an x exists such that $f_k(x) < 0$ for all $k > 1$ (cf. [893], p. 56 ff.).□

Corollary: LP $\max\{x : \mathrm{y}_{\acute{m}} - xy_m = \mathrm{b}_{\acute{m}}, y_n = y_0 = 0, x \leq x_0 \in {}^{\omega}\mathbb{R}\}$ can determine for $m = 1, \ldots, n$ by Horner scheme an $x \in {}^{\omega}\mathbb{R}$ solving $\mathrm{b}_{\acute{n}}\mathrm{x}^{\acute{n}} + \ldots + b_1 x + b_0 = 0$ in $\mathcal{O}(\vartheta^{2})$.□

Set Theory

Definition: Forgoing to distinguish two entities makes them *identical*. The entirety of non-identical entities is called *set* S of *elements* such that $|S|$ is their *number* n. Symbol $\emptyset$ denotes the empty set, which contains no elements. If $S \neq \emptyset$ can be emptied by successively removing half of the remaining elements, which is rounded up until S is empty, then n is *finite*, otherwise *infinite*. Let ν the largest *finite* real number, ω in between the largest *mid-finite* and ι the smallest positive one.△

Conventionally, sufficiently well-understood axioms exist that define the real numbers as a linearly ordered field and the complex numbers as a field with the imaginary unit $\underline{1}$ (read as "comp 1"). Analogously, addition, multiplication, and their inverse operations may be extended to the largest and by definition closed field extensions $\mathbb{R}$, and $\mathbb{C} := \mathbb{R} + \underline{\mathbb{R}}$ (with further operations like exponentiation). Every number from $\mathbb{C}^*$ is said to be *infinitesimal* if the absolute value of its reciprocal is infinite.

Presenting Peano axioms and field axioms is deliberately omitted here to remain clear and to emphasise the new content. Two numbers must have a minimum distance, since *all* different numbers of $\mathbb{R}$ are separated by a distance. Assuming the opposite of the minimum not being fixed leads to the contradiction that every number from $\mathbb{R}$ must have at least one nearest neighbour as a number, which is itself in $\mathbb{R}$ (cf. the isomorphism above).

The set $\mathbb{R}$ of all real numbers is isomorphic to a set of (hyper-) natural or integer numbers. It has both a fixed minimum element and a fixed maximum element, since $\mathbb{R}$ is viewed holistically and completely, from which its closure is obtained together with the field axioms. Otherwise, the former theory would have to be repeatedly adapted to the circumstances. Numbers noted as sequences are plain unwieldy (cf. [741], p. 15 f.).

(Conventionally and infinite natural) induction can show that, starting with the set of conventionally natural numbers, it can be diagonalised up to any power according to Cantor, so that all(!) infinite sets are equipotent to the set of conventionally natural numbers if Hilbert's translations are used as an aid. However, the numbers of the sets of natural and integers differ almost exactly by a factor of 2.

Cantor regarded such sets as of the same cardinal number. His distinction between merely countable and uncountable sets is not differentiated enough. If instead bijections are correctly considered, this gives a different picture. The fuss made so far over ordinal and cardinal numbers is omitted because there is an easier way: the *consistent* extension of the real numbers.

Definition: Successively adding 1 to 0 gives the *set of natural numbers* $\mathbb{N} := \mathbb{N}^* \cup \{0\}$. Numbers from $\underline{\mathbb{R}}^*$ are called *purely complex.* Excluding all composite numbers from $\mathbb{N}_{\geq 2}$ gives the set of *prime numbers* $\mathbb{P}$. The set of *integers* $\mathbb{Z}$ is given by $\mathbb{Z} := \mathbb{N} \cup -\mathbb{N}^*$ and the set of *real numbers* $\mathbb{R}$ by the set of fractions with numerator from $\mathbb{Z}$ and denominator from $\mathbb{N}^*$. The *minimal distance* to 0 is ι. $\mathbb{R}$ includes both the conventionally hyperreal and surreal numbers. The numbers ν, ω and $1/\iota$ have the form 2^n for $n \in \mathbb{N}$.△

Definition: Let $\mathbb{B} = \{0, 1\}$ the *Boolean domain*. *Decrement* and *increment* of $a \in \mathbb{C}$ are given by $\acute{a} := a - 1$ and $\grave{a} := a + 1$ and the reciprocal of $u \in \mathbb{C}^*$ is $\tilde{u} := 1/u$. They are pronounced "a dec", "a inc" and "turn u". Furthermore, $\hat{a} := 2a$, $\breve{a} := a/2$ and $a\text{-} := -a$ are pronounced "a hat", "half a" and "a neg". The logarithm of $a \in {}^{\omega}\mathbb{C} \setminus {}^{\omega}\mathbb{R}_{\leq 0}$ and base b ${}^{\omega}\mathbb{R}_{>0}$ is denoted by ${}_b a$ and pronounced "b log a". If $\sigma \in \{\nu, \omega\}$ and $S^* = S^{*-1}$, let ${}^{\sigma}S := S \cap [-\sigma, \sigma]$ for $S \subseteq \mathbb{R}$, ${}^{\sigma}\mathbb{C} := {}^{\sigma}\mathbb{R} + {}^{\sigma}\underline{\mathbb{R}} \subset \mathbb{C}$ and ${}^{\sigma}S := S \cap {}^{\sigma}\mathbb{C}$ for $S \subseteq \mathbb{C}$.△

Definition: Concerning all $s \in {}^{\sigma}_{\tau}S \subset \mathbb{C}$, the maximum value of their numerators is specified by σ and that of their denominators by τ. The sets $\mathbb{M}_{\mathbb{R}} := {}^{\omega}\mathbb{R} \setminus {}^{\nu}\mathbb{R}$ and $\mathbb{M}_{\mathbb{C}} := \mathbb{M}_{\mathbb{R}} + \underline{\mathbb{M}}_{\mathbb{R}}$ define the *mid-finite* numbers. For real sets, subscript $_{\mathbb{R}}$ may be omitted in the notation. Two sets are *equal* if and only if they contain the same elements (extensionality). The set Y is called *union* of the set X if it contains exactly the elements of the elements of X as elements. Let $\mathcal{P}(X) := \{Y : Y \subseteq X\}$ be the *power set* of the set X.△

Remark: For subsets of S, this holds analogously. For $T := {}^{\sigma}S \notin \{\emptyset, \{0\}\}$, it holds that $T + T \nsubseteq T$ or even $T \cdot T \nsubseteq T$. Sets with leading ${}^{\omega}$ only contain non-infinite elements. Sets ${}^{\nu}S$ are related to the conventional sets S. Mid-finite sets close the gap between finite and infinite ones.

Extreme theorem: Every linear order has exactly both: a maximum and a minimum element since all other constellations lead to a contradiction to totality or to that mentioned above.

Remark: Only the unique construction of a non-finite set allows to determine the number of its elements and to relate it to ${}^{\omega}\mathbb{N}$ as basis thanks to its simple construction. If there are multiple possible constructions, the most plausible one is selected. The relation $\in$ is irreflexive and asymmetric, whereas $\subseteq$ is partial order.

Definition: The sum $\mathrm{p(z)} = +_{k=0}^{\acute{m}} a_k z^k$ for $z \in \mathbb{C}$ and $m \in \mathbb{N}^*$ is called an *m-polynomial*, if the number of *coefficients* with $a_k \in {}^{c}\mathbb{Z}$ or $a_k \in {}^{\omega}\mathbb{Z}$, where $k \in \mathbb{N}_{<m}$ and $a_k \neq 0$, is finite, else *m-series*. Then $\deg(p) := \acute{m}$ for $a_k \neq 0$ is called *degree* of the polynomial or series p. For the *zero polynomial* $p = 0$, $\deg(p) := -1$ holds. The $z \in \mathbb{C}$ setting $\mathrm{p(z)} = 0$ are called *zeros* and are each an *m-algebraic number (AN)*.△

Definition: For $x \in {}^{\omega}\mathbb{R}$ let ${}_{\epsilon^0}(\mathrm{x}) = x$, ${}_{\epsilon^{\grave{n}}}(\mathrm{x}) = {}_{\epsilon}\left({}_{\epsilon^n}(\mathrm{x})\right)$ and ${}_{\epsilon^{\grave{n}}}(\mathrm{x}) := 0$ if ${}_{\epsilon^n}(\mathrm{x}) \leq 1.$. If $p(z)$ is the minimal polynomial of $\alpha \in \mathbb{C}$ for $0 \neq p(\alpha) \approx 0$, let $\min |p(z)| = |p(\alpha)|$. The corresponding sets are denoted $A := {}^{m}\mathrm{A_R}$ in the real case and ${}^{m}\mathrm{R_C} \supset {}^{m}\mathrm{A_C} \supset {}^{m}_{m}\mathrm{R_C} \supset {}^{m}\mathrm{Z_C} \supset {}^{m}\mathrm{N_C}$ in the complex one. For $a_{\deg(p)} = 1$, *m-algebraic integers* are given. The notation for *m*-ANs is $(m; a_{k-1}, a_{k-2}, ..., a_1, a_0; r, i; \#n, \&q; v, p)_s$.△

Definition: For $r \in {}^{\nu}\mathbb{N}^*$ $(-{}^{\nu}\mathbb{N}^*)$, the r-th largest ($|r|$-th smallest) zero exists with real part > 0 (< 0), where $r = 0$, $i \in {}^{\nu}\mathbb{N}^*$ $(-{}^{\nu}\mathbb{N}^*)$ denotes a non-real zero with the i-th largest ($|i|$-th smallest) imaginary part > 0 (< 0), and the other ANs have analogous notations. When at least one a_j is taken as a variable, $\#n$ specifies the number $n \in {}^{\nu}\mathbb{N}^*$ of zeros and $\&q$ gives the number $q \in {}^{\nu}\mathbb{N}$ of repeated zeros. All k-minimal polynomials have the sign $<$ as specification s, all k-minimal series have $>$.△

Remark: Here r takes precedence over i and $r = i = a_0 = 0$ represents the number 0. The numerical value v has the precision p. Not distinguishing between repeated zeros allows the zeros of k-polynomials or k-series with integer coefficients to be endowed with a strict total ordering. The information $r, i; \#n, \&q; v, p$, and s may optionally be omitted as e. g. for fractions. The $(\nu + 2)$-tuple $(0, ..., 0, a_{k-1}, ..., a_0; r, i)_<$ where every $a_j \in {}^{\nu}\mathbb{N}$ gives a strict lexical well-ordering of the ANs.

Examples: The numbers $(\nu; 1, 0, 0, 0, -1)_>$ are $1, -1, \underline{1}$, and $-\underline{1}$. The golden ratio $\Phi := \check{1} + 5^{\tilde{2}}/2$ may be written as $(\nu; 1, -1, -1; 1, 0; 1.618033, 10^{-6})_<$. The number $0.\bar{1} = 0.1...1$ with ω ones after the point is distinct from the number $\tilde{9}$, since $9 \times 0.\bar{1} = 0.9...9 = 1 - 10^{-\omega} \neq 1$. It is therefore not ω-algebraic and should therefore be written as $(\omega, 9 \times 10^{\omega}, 1 - 10^{\omega})$.

Remark: Let $m \in {}^{\nu}\mathbb{N}$ the maximum polynomial degree and $n \in {}^{\nu}\mathbb{N}$ the maximum absolute value that the integer coefficients a_k can take of the polynomials $a_m x^m + a_{\acute{m}} x^{\acute{m}} + ... + a_1 x + a_0$ for $k \in {}^{\nu}\mathbb{N}_{\leq m}$. This makes sense due to the symmetry of the a_k. The number of ANs is the number of zeros of the normalised irreducible polynomials specified by the conditions: greatest common divisor gcd of the coefficients is equal to 1, $a_m > 0$, and $a_0 \neq 0$.

Inclusion theorem: Neither a nonempty set contains itself or its power set, nor it admits a bijection with one of its subsets.

Proof: Every set differs from its elements, since it comprises the latter. Thus $\emptyset \neq \{\emptyset\}$. Its relative complement shows the elements with missing partner element for the bijection.□

Conclusion: Especially, this contradicts Dedekind-infiniteness and Cantor's first diagonal argument, since $\mathbb{N}$ is a proper subset of $\mathbb{R}$. Putting numbers excluded from the beginning to the end refutes the second one. The same is true for the Cantor, the Russell-Zermelo and the Banach-Tarski paradox. A translation of an infinite set always departs from the original set as the successor function s: ${}^{\omega}\mathbb{N} \to {}^{\omega}\mathbb{N}^* \cup \{\grave{\omega}\}$ shows for ${}^{\omega}\mathbb{N}$. This contradicts Hilbert's hotel.

Definition: *Cycle freedom* denotes the absence of cyclic sequences of sets, each containing one as an element in the previous one. *Replaceability* denotes the possible transition from a set X by uniquely replacing each element of X by an arbitrary set. If set Y contains exactly one element from each element of X (postulation of selectability), it is called *selection* of pairwise disjoint nonempty sets from a set X.△

Lemma: Because of $\tilde{v}m \leq 1 \leq$ for all $m \in {}^{\nu}\mathbb{N}$ and $a \in {}^{\nu}\mathbb{R}_{\geq 1}$, the Archimedean axiom is invalid.□

Archimedes' theorem: There exists $m \in {}^{\nu}\mathbb{N}$ such that $a < bm$ if and only if $a < b\nu$ whenever $a > b$ for $a, b \in \mathbb{R}_{>0}$, since $\nu = \max {}^{\nu}\mathbb{N}$ holds.□

Claim: The Cantor polynomial $P(m, n) := \tilde{2}((m + n)^2 + 3m + n)$ bijectively maps ${}^{\omega}\mathbb{N}^2$ to ${}^{\omega}\mathbb{N}$.

Refutation: It holds that $P(\omega, \omega) = \hat{\omega}\dot{\omega} > \omega = \max {}^{\omega}\mathbb{N}$.□

Remark: Similarly, the Fueter-Pólya conjecture is refuted. If the set ${}^{\omega}\mathbb{N}^2$ is replaced by $\{(m, n) \in {}^{\omega}\mathbb{N}^2 : m + n \leq k \in {}^{\omega}\mathbb{N}\}$ for $k(k + 3) = \hat{\omega}$, the claim holds. Since infinitely many sets exist whose number of elements lie between $|{}^{\nu}\mathbb{N}|$ and $|{}^{\nu}\mathbb{R}|$, the continuum hypothesis is wrong, too.

Foundation theorem: Putting $X := \{x_m : x_0 := \{\emptyset\}, x_\omega := \{x_1\}$ and $x_{n+1} := \{x_n\}$ for $m \in {}^{\omega}\mathbb{N}$ and $n \in {}^{\omega}\mathbb{N}_{\geq 2}\}$, only postulating the axiom of foundation guarantees cycle freedom that every nonempty subset $X \subseteq Y$ contains an element x_0 such that X and x_0 are disjoint.□

Remark: Setting $x_\omega := \{x_0\}$ instead of $x_\omega := \{x_1\}$, X becomes an infinite chain. All definitions above determine the set theory represented here, which does not require proper classes since only meaningful sets are considered. The existing set $\mathbb{W}$ of all worlds cannot be changed, because otherwise it would contradict completeness. However, this does not mean that changes in the worlds do not exist or that everything is determined. It helps to look at time spatially.

Definition: Let $\mathbb{K} \in \{\mathbb{C}, \mathbb{R}\}$. Two different points x and y in a subset $M \subseteq \mathbb{K}^n$ where $n \in \mathbb{N}^*$ are said to be *neighbours* if $\|x\text{-}y\| \leq \max(\|x\text{-}z\|, \|y\text{-}z\|)$ holds for all points $z \in M$, where $\|\cdot\|$ denotes the Euclidean norm. If all neighbouring points of $M \subseteq \mathbb{K}^n$ have the minimal distance $h \in \mathbb{R}_{>0}$, M is said to be *h-homogeneous* and is denoted by h-M. The case $h = \iota$ makes M *gapless*. It holds the isomorphism $\mathbb{C}^n \cong \mathrm{R}^{\hat{\mathrm{n}}}$.△

Definition: Let $u1_n := (u, \ldots, u)^{\mathrm{T}} \in \mathbb{K}^n$. Read $u1_n$ as "u rep n". A subset $M \subseteq \mathbb{K}^n$ is said to be *dense* in $\mathbb{K}^n$ if there is a point $y \in M$ for every $M \subseteq \mathbb{K}^n$ with $\|x\text{-}y\| = \iota$. For the point-symmetric $\dot{\mathrm{K}}^{\mathrm{n}}$ related to $\mathbb{K}^n$, an *n-cube* with edge length σ is given by ${}^{\sigma}\mathbb{R}^n := \bigtimes_{k=1}^{n} {}^{\sigma}\mathbb{R}$ and an *n-ball* by ${}^{\sigma}\dot{\mathrm{R}}^n$ with radius σ. Let $\mathbb{U}$ the *fuzzy set*, $\mathbb{Q}$ the set of *quaternions*, $\mathbb{O}$ the set of *octonions* and $\mathbb{S}$ the set of *sedenions*.△

Remark: To h-homogenise a set, move h away from the origin in each dimension and round elements in between to the nearest h-homogeneous elements. The maximum number of leading and also fractional digits of elements of $\tilde{\nu}$-${}^{\nu}\mathbb{R}$, $\tilde{\omega}$-${}^{\omega}\mathbb{R}$ and (ι-)$\mathbb{R}$ is given by the logarithms to base 2 (see Nonstandard Analysis) ${}_2\nu$, ${}_2\omega$ and ${}_2\tilde{\iota}$.

Fundamental theorem of set theory: The set $\mathbb{R}$ is a maximal, linearly ordered, closed, continuous and ι-homogeneous field giving $|\mathbb{R}| = 2\tilde{\iota}^2 + 1$, since h-homogenised elements with distance ι guarantee continuity, although the exact value of ι is undetermined.□

Remark: Therefore, irrational numbers do not exist and $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$ as well as $\mathbb{C}$ are h-homogeneous, which solves the Erdős-Ulam problem. Since reconstructing real numbers is unambiguous with necessarily periodic fractional expansion, h-homogeneity does not restrict significantly. A complete homogeneity (i. e. in all directions) of higher-dimensional real spaces is unfortunately impossible, as a simple consideration of circles or balls shows – also when one-dimensional spaces are rolled up.

Conclusion: A linear order can be defined for every non-empty set.□

Remark: Unlike the conventional way, immediate predecessors and successors can be specified. Vividly, this is a threading of pearls into a chain where the number of dimensions does not matter. Individual points from $\mathbb{R}^n$ can be linearly ordered with $n \in \mathbb{N}^*$, for example, dimension by dimension: The total chain length is then $\iota\,|\mathbb{R}|^n$. Another linear order is the spiral order starting at 01_n according to the Euclidean norm for individual points.

Remark: Only malicious gossip has it that is wrong what is not trivially true in Cantor's set theory. What is certain, however, is that much of the latter cannot be maintained so far, but still contains enough interesting ideas to be pursued. A more detailed evaluation is omitted here for reasons of fairness.

Nonstandard Analysis

Preliminary remarks: In the following section, the definitions established in the chapters on Set Theory and Topology are used, and usually take $m, n \in {}^{\omega}\mathbb{N}^*$. Integration and differentiation are studied on an arbitrary non-empty subset A from ${}^{(\omega)}\mathbb{K}^n$. The mapping concept requires replacing every element not in the image set by the neighbouring element in the target set. If multiple choices are possible, one single choice is selected. The following may be easily generalised to other sets and norms.

Definition: The function $||\cdot||: \mathbb{V} \to {}^{(\omega)}\mathbb{R}_{\geq 0}$ where $\mathbb{V}$ is a vector space over ${}^{(\omega)}\mathbb{K}$ is called a *norm*, if for all $x, y \in \mathbb{V}$ and $\lambda \in {}^{(\omega)}\mathbb{K}$, it holds that: $\|x\| = 0 \Rightarrow x = 0$ (*definiteness*), $\|\lambda x\| = |\lambda|\|x\|$ (*homogeneity*), and $\|x + y\| \leq \|x\| + \|y\|$ (*triangle inequality*). The *dimension* of $\mathbb{V}$ is given by the maximal number of linearly independent vectors and is denoted by $\dim \mathbb{V}$. The norms $||\cdot||_a$ and $||\cdot||_b$ are said to be *equivalent* if there exist $s, t \in [\tilde{\nu}, \nu]$ such that $s\|x\|_b \leq \|x\|_a \leq t\|x\|_b$ for all $x \in \mathbb{V}$. Let N be the set of all norms in $\mathbb{V}$.△

Theorem: Norms are equivalent if and only if $||x||_a/||x||_b \in [\tilde{\nu}, \nu]$ for all $||\cdot||_a, ||\cdot||_b \in N$ and all $x \in \mathbb{V}^*$ by putting $s := \min\{||x||_a/||x||_b : x \in \mathbb{V}^*\}$ and $t := \max\{||x||_a/||x||_b : x \in \mathbb{V}^*\}$.□

Minimality theorem: For every $r \in \mathbb{R}$ with $n \in \mathbb{N}$ positions after the floating point, the geometric series (GS) yields a unique b-adic expansion and $\min\{b \in \mathbb{R}_{>1}\} = 2$, where $(1 - b^{-n})/(1 - b^{-1}) < 2$ holds.□

Remark: For $b \in]1, 2[$, the uniqueness is lost and two digits (0 and 1), intended for the dual representation, must be used. *One* digit only makes sense for the base $b = 1$.

Definition: For the *characteristic function* χ, let $\chi_A(a) := 1$ for $a \in A$ and $\chi_A(a) := 0$ for $a \notin A$. The set ${}^{:}\mathbb{R} := \mathbb{R} \cup \{-\infty, \infty\}$ allows calculations with $\infty \gg \tilde{\iota}^2$ like having a constant. Let $\operatorname{sgn}(z) := \tilde{z}|z|\chi_{\mathbb{C}^*}(z)$ for complex z and $\operatorname{sgn}(x) := \tilde{x}|x|\chi_{:\mathbb{R}^*}(x)$ for real x. The area of ${}^{1}\dot{\mathbb{R}}^2$ gives *pi* π. *Euler's number* ϵ (read briefly as "eps") is defined as the solution of $x^{\pi} = -1$. Then the *logarithm function* ln is defined by $\epsilon^{\ln z} = z$ and the *power function* by $z^s = \epsilon^{s \ln z}$ for $s, z \in \mathbb{C}$. This allows giving a formal definition of *exponentiation*.△

Remark: If ± 0 is replaced by $\pm\tilde{\infty}$, calculations become unique and consistent. The definition of ϵ is $\mathcal{O}(\tilde{\nu})$ larger than that by $(1 + \tilde{\nu})^{\nu}$ (calculate with approximations!). The exponential series justifies the former when being exactly differentiated with as many terms as possible.

Definition: The function $\mu_h: A \to \mathbb{R}_{\geq 0}$ where $A \subseteq {}^{(\omega)}\mathbb{C}^n$ is an m-dimensional set with $h \in \mathbb{R}_{>0}$ less than or equal the minimal distance of the points in A, $m \in {}^{\omega}\mathbb{N}_{\leq \hat{n}}$, $\mu_h(A) := |A|\ h^m$ and $\mu_h(\emptyset) = |\emptyset| = 0$ is called the *exact h-measure* of A being *h-measurable*. Let the *exact standard measure* be μ_ι (ι may be omitted).△

Remark: Answering positively the measure problem, the union A of pairwise disjoint h-homogeneous sets A_i for $i \in I \subseteq \mathbb{N}$ clearly additively and uniquely results in $\mu_h(A) = \bigplus_{i \in I} \mu_h(A_i)$. Its strict monotony follows for h-homogeneous sets $A_1, A_2 \subseteq {}^{(\omega)}\mathbb{K}^n$ satisfying $A_1 \subset A_2$ from $\mu_h(A_1) < \mu_h(A_2)$. If h is not equal on all considered sets A_i, the minimum of all h is chosen and the homogenisation follows as described in Set Theory. In the following, let $||\cdot||$ the Euclidean norm.

Examples: Consider the set $A \subset [0, 1[^n$ of points, whose least significant bit is 1 (0) in all $n \in {}^{\omega}\mathbb{N}^*$ coordinates. Then $\mu_\iota(A) = \tilde{2}^n$. Since A is an infinite and conventionally uncountable union of individual points without the neighbouring points of $[0, 1[^n$ in A, and these points are Lebesgue null sets, A is not Lebesgue measurable, however it is exactly measurable. Domains from ${}^{(\omega)}\mathbb{K}^n$ that are more densely pushed together have no smaller (larger) intersection (union) than previously.

Remark: The exact h-measure is optimal, since it only considers the neighbourhood relations of points, i. e. in the extreme case distances of points parallel to the coordinate axes. Concepts such as σ-algebras and null sets are dispensable, since the only null set is the empty set $\emptyset$.

Definition: Neighbouring points in A are described by the irreflexive symmetric *NR* $B \subseteq A^2$. The function $\gamma: C \to A \subseteq \mathbb{C}^n$, where $C \subseteq \mathbb{R}$ is h-homogeneous and h is infinitesimal, is called a *path* if $\|\gamma(x) - \gamma(y)\|$ is infinitesimal for all neighbouring points $x, y \in C$ and $(\gamma(x), \gamma(y)) \in B$. Let $z_0 \in A \subseteq \mathbb{K}^n$ and $f: A \to {}^{(\nu)}\mathbb{K}^m$. NRs are systematically written as (predecessor, successor) with the notation $(z_0, \vec{z}_0)$ or $(\overleftarrow{z}_0, z_0)$, pronouncing $\rightharpoonup$ as "post" and $\leftharpoonup$ as "pre".△

Definition: If $\|f(\vec{z}_0) - f(z_0)\| < \alpha$ for infinitesimal $\alpha \in {}^{(\omega)}\mathbb{R}_{>0}$, f is defined *α-successor-continuous* in z_0 in the direction $\vec{z}_0$. If the exact modulus of α does not matter, α may be omitted in the notation. If f is α-successor-continuous for all z_0 and $\vec{z}_0$, it simply is called α-continuous. It holds that α is the *degree* of continuity. If the inequality only holds for $\alpha = \tilde{v}$, f simply is called (successor-) continuous.△

Example: The function $f\colon \mathbb{R} \to \{\pm 1\}$ with $f(x) = \underline{1}^{\tilde{\iota}\hat{x}}$ is nowhere successor-continuous on $\mathbb{R}$, but its modulus is (cf. Number Theory). Here, $\tilde{\iota}x$ is an integer since $\mathbb{R}$ is ι-homogeneous. Setting $f(x) = 1$ for finite fractions x and $= -1$ otherwise, then $f(x)$ is partially ι-successor-continuous on infinite fractions x, unlike the conventional notion of continuity.

Example of a Peano curve (cf. [739], p. 188): Consider the even, periodic $g\colon \mathbb{R} \to \mathbb{R}$ with period 2 and image $I := [0, 1]$ given by $g(s) = \chi_{[1,\check{3}]}(\tilde{s}) + \chi_{]\check{3},3[}(\tilde{s})(3s - 1)$. Let the function $\phi\colon I \to \mathbb{R}^2$ be defined by

$$\phi(s) = \left(+_{n=0}^{\omega} \tilde{2}^{\grave{n}} g\left(4^{\hat{n}} s\right), +_{n=0}^{\omega} \tilde{2}^{\grave{n}} g\left(4^{\hat{n}+1} s\right)\right).$$

The function ϕ is at least continuous since the sums are ultimately locally linear functions in s. It would however be an error to believe that I can be bijectively mapped onto I^2 in this way: the powers of four in the function g, and the values 0 and 1 taken by g in two sub-intervals thin out I^2 so much that a bijection is clearly impossible. Restricting the proof to finite fractions only is simply insufficient.

Definition: For $f\colon A \to {}^{(\omega)}\mathbb{K}^m$, $\downarrow f(\vec{z}) := f(\vec{z}) - f(z)$ is called *successor-differential* of f in the direction $\vec{z}$ for $z \in A$. If $\dim A = n$, then $\downarrow f(\vec{z})$ stands for a successor-derivative in every variable. Mixed differentials are specified by several arrows. If $f(z) = z$, then $\downarrow \vec{z}$ can be written instead of $\downarrow f(\vec{z})$. Clear A or B remain unmentioned.

Definition: Read ↓ as "down". If $|f(\vec{x}) - f(x)| > \tilde{\omega}$ holds for x of $f\colon A \subseteq {}^{\omega}\mathbb{R} \to {}^{\omega}\mathbb{R}$, x is called a *jump discontinuity*. If the modulus of the successor-differential of f in the direction $\vec{z}$ at $z \in A$ is smaller than α and infinitesimal, then f is also rated as α-successor-continuous there. A function $f\colon A \subseteq {}^{(\omega)}\mathbb{K}^n \to {}^{\omega}\mathbb{R}$ is said to be *convex (concave)* (read $f \in Con(A)$) if the line segment between any two points on the graph of the function lies above (below) or on the graph. Let it *strictly* convex (concave) if "or on" can be omitted.△

Theorem: For $A \subseteq {}^{(\omega)}\mathbb{K}^n$, all functions $f \in Con(A)$ are α-successor-continuous and successor-differentiable.□

Definition: The m arithmetic means of all $f_k(\vec{z})$ of $f(z)$ give the m *averaged normed tangential normal vectors* of m unique hyperplanes, defining the mn continuous derivatives of the Jacobian matrix of f, which is not necessarily continuous. The hyperplanes are taken to pass through $f_k(\vec{z})$ and $f(z)$ translated towards 0. A quite simple linear programme (cf. Linear Programming) minimises the moduli of their coefficients.△

Definition: The *derivative* in the direction $\vec{z}_k$ of $F\colon A \to {}^{(\omega)}\mathbb{K}$ at $z = (z_1, \ldots, z_n) \in A \subseteq {}^{(\omega)}\mathbb{K}^n$ with $k \in \mathbb{N}^*_{\leq n}$ is defined as

$$\frac{\downarrow F(\vec{z})}{\downarrow \vec{z}_k} := \frac{F(z_1, \ldots, \vec{z}_k, \ldots, z_n) - F(z)}{\vec{z}_k - z_k}.$$

Definition: The derivative of a function $f\colon A \to {}^{(\omega)}\mathbb{R}$, where $A \subseteq {}^{(\omega)}\mathbb{R}$, is defined to be 0 if and only if 0 lies in the interval defined by the boundaries of the exact predecessor and successor derivative or where f is discontinuous. Let $\mathrm{num}(x) = p \in \mathbb{Z}$ the *numerator function* and $\mathrm{den}(x) = |q| \in \mathbb{N}^*$ the *denominator function* of $x = p/q \in \mathbb{R}$ for coprime p and q (in short: $p \perp q$).△

With this notation, if the function f satisfies $f = (f_1, \ldots, f_n)\colon A \to {}^{(\omega)}\mathbb{K}^n$ with $z \in A \subseteq {}^{(\omega)}\mathbb{K}^n$

$$f(\vec{z}) = \left(\frac{F(\vec{z}_1, z_2, \ldots, z_n) - F(z_1, \ldots, z_n)}{\vec{z}_1 - z_1}, \ldots, \frac{F(z_1, \ldots, z_{n-1}, \vec{z}_n) - F(z_1, \ldots, z_n)}{\vec{z}_n - z_n}\right) = \left(\frac{\downarrow F_1(\vec{z})}{\downarrow \vec{z}_1}, \ldots, \frac{\downarrow F_n(\vec{z})}{\downarrow \vec{z}_n}\right),$$

then $f(\vec{z}) = \nabla F(\vec{z})$ with the *Nabla operator* ∇ is said to be *exact successor-derivative* ${}^1F(\vec{z})$ or the *exact successor-gradient* $\mathrm{grad}\, F(\vec{z})$ of the function $F(z)$, which is said to be *exactly differentiable* at z in direction $\vec{z}$.

If this definition is satisfied for every $z \in A$, then F is said to be an *exactly differentiable antiderivative (AD)* of f. If all directions have the same value, *holomorphicity* is obtained ($A = {}^{v}\mathbb{C}$ and $n = 1$ make F holomorphic). On a domain $\mathbb{D}$, let $\mathcal{O}(\mathbb{D}) \subseteq \mathcal{C}(\mathbb{D}) \subseteq \mathbb{C}$ be the *ring of holomorphic* resp. *continuous functions*.△

Chain rule: For $x \in A \subseteq {}^{(\omega)}\mathbb{R}$, $B \subseteq A^2$, $f\colon A \to C \subseteq {}^{(\omega)}\mathbb{R}$, $D \subseteq C^2$, $g\colon C \to {}^{(\omega)}\mathbb{R}$, choosing $g(\vec{x}) = \vec{g}(x)$, it holds that:

$$ {}^1f\big(g(x)\big) = {}^1f\big(g(x)\big)\,{}^1g(x). $$

Proof:

$$ {}^1f\big(g(x)\big) = \frac{f\big(g(\vec{x})\big) - f\big(g(x)\big)}{g(\vec{x}) - g(x)}\,\frac{g(\vec{x}) - g(x)}{\vec{x} - x} = \frac{f\big(\vec{g}(x)\big) - f\big(g(x)\big)}{\vec{g}(x) - g(x)}\,g'(x) = {}^1f\big(g(x)\big)\,{}^1g(x). \square $$

Product rule: Adding and subtracting $f(\vec{x})g(x)$ and $f(x)g(\vec{x})$ in the numerator yields

$$ {}^1(fg)(x) = {}^1f(x)g(x) + f(\vec{x})\,{}^1g(x) = {}^1f(x)g(\vec{x}) + f(x)\,{}^1g(x)\,. \square $$

Quotient rule: The same for $f(x)g(x)$ and $f(\vec{x})g(\vec{x})$ yields for denominators $\neq 0$ of the following quotients

$$ {}^1\left(\frac{f}{g}\right)(x) = \frac{{}^1f(x)g(x) - f(x)\,{}^1g(x)}{g(x)g(\vec{x})} = \frac{{}^1f(x)g(\vec{x}) - f(\vec{x})\,{}^1g(x)}{g(x)g(\vec{x})}\,. \square $$

Remark: Arguments and function values must belong here to a smaller level of infinity than $\tilde{\iota}$, and f and g must be sufficiently (α-) continuous at $x \in A$. I. e. α must be sufficiently small to allow $\vec{x}$ to be replaced by x. An analogous principle holds for infinitesimal arguments. The exact derivative of the reciprocal function reads ${}^1f^{-1}(y) = 1/{}^1f(x)$ from $y = f(x)$ and identity $x = f^{-1}(f(x))$ by the chain rule and the same precision. L'Hôpital's rule makes sense for (α-) continuous functions f and g, and follows for $f(v) = g(v) = 0$ where $v \in A$ and $g(\vec{v}) \neq 0$ from

$$ \frac{f(\vec{v})}{g(\vec{v})} = \frac{f(\vec{v}) - f(v)}{g(\vec{v}) - g(v)} = \frac{{}^1f(\vec{v})}{{}^1g(\vec{v})}. $$

Remark: Wherever the quotient is defined in the (conventional) (infinite) real case, put alternatively ${}^1F(\vec{v}) \coloneqq f(\vec{v}) - f(\tilde{v})/(\vec{v} - \tilde{v})$. This is especially useful when $\vec{v} - v = v - \tilde{v}$, and the combined derivatives both have the same sign. This definition has the advantage viewing ${}^1F(\vec{v})$ as the "tangent slope" at the point v, especially when F is α-continuous at v. This can be extended to the (conventional) complex numbers analogously.

Definition: Given $z \in A \subseteq {}^{(\omega)}\mathbb{K}^n$, it is $\uparrow_{z\in A} f(z)\downarrow z \coloneqq +_{z\in A} f(z)(\vec{z} - z)$ called the *exact successor integral* of the *vector field* $f = (f_1, ..., f_n)\colon A \to {}^{(\omega)}\mathbb{K}^n$ on A and $f(z)$ is said to be *exactly integrable*. If this requires removing at least one point from A, then the exact integral is called *improper*. Read ↑ as "up". For γ: $G = [a, b[\cap C \to A \subseteq {}^{(\omega)}\mathbb{K}^n$, $C \subseteq \mathbb{R}$, and $f = (f_1, ..., f_n)\colon A \to {}^{(\omega)}\mathbb{K}^n$, $\uparrow_\gamma f(\zeta)\downarrow\zeta = \uparrow_{s\in G} f\big(\gamma(s)\big)\,{}^1\gamma(s)\downarrow s$ where $\downarrow s > 0$, $\vec{s} \in]a, b] \cap C$, choosing $\vec{\gamma}(s) = \gamma(\vec{s})$, since $\zeta = \gamma(s)$ and $\downarrow\zeta = \gamma(\vec{s}) - \gamma(s) = {}^1\gamma(s)\downarrow s$ (i.e. for $C = \mathbb{R}$, B maximal in ${}^{\omega}\mathbb{C}^2$, and D maximal in ${}^{\omega}\mathbb{R}^2$), is called the *exact line-integral (LI) of the vector field* f along the path γ. Improper exact LIs are founded analogously to exact integrals, except that only interval end points may be removed from G. Read $u\restriction_k := u_k$ in $(u_1, ..., u_n) \in {}^{\omega}\mathbb{K}^n$ as "u proj k".△

Theorem, improving Froda's one: A monotone function $f\colon [a, b] \to {}^{\omega}\mathbb{R}$ has at most $2\omega^2 - 1$ jump discontinuities, since at most $2\omega^2$ jump discontinuities are possible between $-\omega$ and ω, with a jump of $\widetilde{\omega}$ if the function does not decrease at non-discontinuities, like a step function.□

Laisant's theorem: For $c \in {}^{\omega}\mathbb{R}$, the product rule yields $\uparrow f(x) \downarrow x|_{f^{-1}(y)} = \uparrow y\frac{\downarrow x}{\downarrow y}\downarrow y = yf^{-1}(y) - \uparrow f^{-1}(y)\downarrow y + c$.□

Remark: The (linear) exact LI on ${}^{(\nu)}\mathbb{K}$ does not need a continuous f, exists always and is usually consistent with the conventional LI. It is monotone in the (conventional) (infinite) real case.

Intermediate value theorem: Let $f\colon [a, b] \to {}^{(\omega)}\mathbb{R}$ α-continuous in $[a, b]$. Then $f(x)$ takes for $x \in [a, b]$ every value between $\min f(x)$ and $\max f(x)$ with precision $< \alpha$. If f is continuous in ${}^{\omega}\mathbb{R}$, it takes every value of ${}^{\nu}\mathbb{R}$ between $\min f(x)$ and $\max f(x)$.

Proof: A gapless chain of overlapping α-environments exists between $\min f(x)$ and $\max f(x)$ where $f(x)$ is centre, since otherwise there would be a contradiction to the α-continuity of f. The second part of the claim follows from the fact that a deviation $|f(\vec{x}) - f(x)| < \tilde{v}$ in ${}^{\nu}\mathbb{R}$ falls below the conventional resolution maximally permitted.□

Remark to the extreme value theorem: The continuous function $f(x) := \widehat{\omega}\sin(\omega x)$ attains for $x \in [-1, 1]$ the minima $-\widehat{\omega}$ and the maxima $\widehat{\omega}$ as infinite values.

Example: The $\hat{\iota}$-continuous function f: ${}^{(\omega)}\mathbb{R} \to \{0, \iota\}$ defined by $f(x) := \check{\iota}(\underline{1}^{\hat{x}\hat{\iota}} + 1)$ consists of only the local minima 0 and the local maxima ι, and has the left and right exact derivatives ± 1.

Examples: In ([763], p. 160), it holds that $r_1 = r_2 = 3$. For $q = \mathrm{den}(x)$ and $f(x) := \chi_{\omega_{\mathbb{R}}}(x)\underline{1}^{\hat{q}}\tilde{q}\dot{q}$, it has f: $[0, 1] \to [\dot{\iota}, -\dot{\iota}]$ the two relative extrema $\pm\dot{\iota}$ (cf. ib. p. 24).

Definition: For all $x \in V$ of an h-homogeneous n-volume $V \subseteq [a_1, b_1]\times...\times[a_n, b_n] \subseteq {}^{(\omega)}\mathbb{R}^n$ with $B = B_1\times...\times B_n$, $B_k \subseteq [a_k, b_k]^2$ and $|\downarrow x_k| = h$ for all $k \in \mathbb{N}^*_{\leq n}$, such that $f(x) := 0$ for all $x \in {}^{(\omega)}\mathbb{R}^n \setminus V$,

$$\uparrow_{x\in V} f(x)\downarrow x \coloneqq \uparrow_{x\in V} f(x)\downarrow(x_1, \dots, x_n) \coloneqq \uparrow_{a_n}^{b_n} \dots \uparrow_{a_1}^{b_1} f(x)\downarrow x_1 \dots \downarrow x_n$$

is called the *exact volume integral* of the *volume integrable* function f: ${}^{(\omega)}\mathbb{R}^n \to {}^{(\omega)}\mathbb{R}$.△

Remark: The isomorphy of $\mathbb{C}$ and $\mathbb{R}^2$ provides something similar for the complex case and $\uparrow_{x\in V} \downarrow x \coloneqq \mu_h(V)$.

Example: Using the exact volume integral in contrast to the Lebesgue integral,

$$\|f\|_p \coloneqq (\uparrow_{x\in V}\|f(x)\|^p\downarrow x)^{\tilde{p}}$$

satisfies for arbitrary f: ${}^{(\omega)}\mathbb{R}^n \to {}^{(\omega)}\mathbb{R}^m$ and $p \in [1, \omega]$ all the properties of a norm, also definiteness.

Example: Let $[a, b[\cap h^\omega\mathbb{Z} \neq \emptyset$ be an h-homogeneous subset of $[a, b[\cap {}^\omega\mathbb{R}$, and write $B \subseteq [a, b[\cap h^\omega\mathbb{Z} \times]a, b] \cap h^\omega\mathbb{Z}$. Now let T be an AD of a not necessarily convergent Taylor series (TS) t on $[a, [\cap h^\omega\mathbb{Z}$ and define $f(x) := t(x) + \varepsilon\, \underline{1}^{\hat{x}/h}$ for conventionally real x and $\varepsilon \geq \tilde{v}$. For $h = \tilde{v}$, f is nowhere continuous, and thus is conventionally nowhere differentiable or integrable on $[a, b[\cap h^\omega\mathbb{Z}$, but for all h holds

$${}^1f(x) = t'(x) - \widetilde{\downarrow x}\hat{\varepsilon}\underline{1}^{\hat{x}/h}$$

and

$$\uparrow_{x\in[a,b[\,\cap\, h^\omega\mathbb{Z}} f(x)\downarrow x = T(b) - T(a) + \check{\varepsilon}\left(\underline{1}^{\hat{a}/h} - \underline{1}^{\hat{b}/h}\right).$$

Example: The conventionally non-measurable middle-thirds Cantor set $C_{\tilde{3}}$ has measure $\mu_\iota(C_{\tilde{3}}) = \check{3}^{-\omega}$. Consider the function c: $[0, 1] \to \{0, \check{3}^\omega\}$ defined by $c(x) = \check{3}^\omega\chi_{C_{\tilde{3}}}$. Then

$$\uparrow_{x\in[0,1]} c(x)\downarrow x = +_{x=0}^{1} c(x)\downarrow x = \check{3}^\omega\mu_\iota(C_{\tilde{3}}) = 1.$$

Definition: A *sequence* (a_k) with *members* a_k is a mapping from ${}^{(\omega)}\mathbb{Z}$ to ${}^{(\omega)}\mathbb{C}^m$: $k \mapsto a_k$. A *series* is a sequence (s_k) with $m \in {}^{(\omega)}\mathbb{Z}$, *radius of convergence* r and *partial sums* $s_k = +_{j=m}^{k} a_j$. A sequence (a_k) with $k \in {}^{(\omega)}\mathbb{N}^*$, $a_k \in {}^{(\omega)}\mathbb{C}$ and $\alpha \in]0, \tilde{v}]$ is called *α-convergent* to $a \in {}^{(\omega)}\mathbb{C}$ if there exists $m \in {}^{(\omega)}\mathbb{N}^*_{\leq k}$ where $|a_k - a| < \alpha$ for all a_k such that $k - m$ is not too small. The set α-A of all such a is called *set of α-limit values* of (a_k). A uniquely determined representative of this set (e. g. the final value or mean value) is called the *α-limit value* α-a. For the case $a = 0$, the sequence is called a *zero sequence*. If the inequality only holds for $\alpha = \tilde{v}$, the α- is omitted. Usually, k will be chosen maximal and α minimal.△

Remark: Conventional limit values are hardly more precise than $\mathcal{O}(\tilde{\omega})$. Their actual rationality or pure algebraicity is seldom regarded! To avoid the exclusive relevance of the largest index of each sequence (cf. [813], p. 144), the conventional definition requires the completion that infinitely many or almost all members of the sequence have an arbitrarily small distance from the limit value. Only finitely many may have a larger distance. Then only the monotone convergence is valid (cf. loc. cit., p. 155).

Remark: The fundamental theorem of set theory makes the representation of each positive number by a determined, unique, infinite decimal fraction baseless (cf. p. 27 f.). Putting $\varepsilon := \iota$ any proof claiming that, for $\varepsilon \in {}^{(\omega)}\mathbb{R}_{>0}$ – especially for all $\varepsilon \in {}^{(v)}\mathbb{R}_{>0}$ – there exists a real number $\varepsilon\tilde{r}$ with real $r \in {}^{(\omega)}\mathbb{R}_{>1}$, is false. Otherwise, an infinite regression may occur. The εδ-definition of the limit value (it is questionable that δ exists, p. 235 f.) requires ε as a specific multiple of ι making that of continuity also true (see p. 215).

Remark: Consider for example the real function that doubles every real value but is not even uniformly continuous. Uniform continuity need not be considered, since in general $\delta := \iota$ and ε accordingly larger. If

two function values do not satisfy the conditions, then the function is not continuous at that point. Thus, continuity is equivalent to uniform continuity, by choosing the largest ε from all admissible infinitesimal values. Easily, continuity is equivalent to Hölder continuity.

Remark: Here infinite real constants may be allowed. The same is true for uniform convergence, since the maximum of the indices may be chosen such that each argument as the index satisfies everything, and $\acute{\omega}$ is sufficient in every case. Otherwise, pointwise convergence also fails. Thus, uniform convergence is equivalent to pointwise convergence, by choosing the largest of all admissible infinitesimal values.

Fubini's theorem: For $X, Y \subseteq {}^{(\omega)}$ and $f\colon X{\times}Y \to {}^{(\omega)}\mathbb{K}$, a reordering of integral sums shows

$$\uparrow_Y\uparrow_X f(x,y)\downarrow x\downarrow y = \uparrow_{X\times Y} f(x,y)\downarrow(x,y) = \uparrow_X\uparrow_Y f(x,y)\downarrow y\downarrow x. \square$$

Example: By the principle of latest substitution (see below), $r_\pm^2 \coloneqq x^2 \pm y^2$ results for

$$\uparrow_{[a,b[\times[c,d[}\ \tilde{r}_+^4 r_-^2\downarrow(x,y) = \uparrow_a^b \tilde{r}_+^2 y\downarrow x|_c^d = -\uparrow_c^d \tilde{r}_+^2 x\downarrow y|_a^b = \arctan\tfrac{d}{b} - \arctan\tfrac{c}{b} + \arctan\tfrac{d}{a} - \arctan\tfrac{c}{a}$$

in the (improper) integral

$$I(a,b) \coloneqq \uparrow_{[a,b[^2}\ \tilde{r}_+^4 r_-^2\downarrow(x,y) = \arctan\tfrac{b}{b} - \arctan\tfrac{a}{b} + \arctan\tfrac{b}{a} - \arctan\tfrac{a}{a} = \breve{\pi} - \breve{\pi} = 0$$

and not

$$I(0,1) \coloneqq \uparrow_0^1\uparrow_0^1 \tilde{r}_+^4 r_-^2\downarrow y\downarrow x = \uparrow_0^1 \tfrac{\downarrow x}{1+x^2} = \tfrac{\pi}{4} \neq \tfrac{\pi}{4} = -\uparrow_0^1 \tfrac{\downarrow y}{1+y^2} = \uparrow_0^1\uparrow_0^1 \tilde{r}_+^4 r_-^2\downarrow x\downarrow y = I(0,1).$$

Exchange theorem: (Transfinite) induction shows the result of multiple derivatives of a function $f\colon A \to {}^{(\omega)}\mathbb{K}$ as independent of the order of differentiation, provided that variables are only evaluated and limits are only computed at the end, if applicable (*principle of latest substitution*).□

Example: For $f\colon {}^{\omega}\mathbb{R}^2 \to {}^{\omega}\mathbb{R}$, $f(0,0) = 0$ and $f(x,y) = \tilde{r}_+^2 xy^3$ where $r_\pm^2 \coloneqq x^2 \pm y^2$, it holds that

$$\frac{\downarrow^2 f}{\downarrow x\downarrow y} = \tilde{r}_+^2(y^6 + 6x^2y^4 - 3x^4y^2) = \frac{\downarrow^2 f}{\downarrow y\downarrow x}$$

with value $\tilde{2}$ at the point $(0,0)$, even though having y for $x = 0$ on the left and 0 on the right for $y = 0$ in

$$\frac{\downarrow f}{\downarrow x} = -\tilde{r}_+^4 r_-^2 y^3 \neq \tilde{r}_+^4(xy^4 + 3x^3y^2) = \frac{\downarrow f}{\downarrow y},$$

where differentiating with respect to the other variable gives on the left $1 \neq 0$ on the right.

First fundamental theorem of exact differential and integral calculus for *LI*s: The function $F(z) = \uparrow_\gamma f(\zeta)\downarrow\zeta$ where $\gamma\colon [d, x[\ \cap C \to A \subseteq {}^{(\omega)}\mathbb{K}$, $C \subseteq \mathbb{R}$, $f\colon A \to {}^{(\omega)}\mathbb{K}$, $d \in G = [a, b[\ \cap C$, and choosing $\vec{\gamma}(x) = \gamma(\vec{x})$, is exactly differentiable, and ${}^1F(z) = f(z)$ holds for all $x \in G$ and $z = \gamma(x)$.

Proof:
$$\begin{aligned}\downarrow F(z) &= \uparrow_{s\in[d,x]\,\cap\, C} f(\gamma(s))\,{}^1\gamma(s)\downarrow s - \uparrow_{s\in[d,x[\,\cap\, C} f(\gamma(s))\,{}^1\gamma(s)\downarrow s = \uparrow_x f(\gamma(s))\tfrac{\gamma(\vec{s})-\gamma(s)}{\vec{s}-s}\downarrow s\\ &= f(\gamma(x))\,{}^1\gamma(x)\downarrow x \quad = \quad f(\gamma(x))(\vec{\gamma}(x) - \gamma(x)) \quad = \quad f(z)\downarrow z. \square\end{aligned}$$

Second fundamental theorem of exact differential and integral calculus for *LI*s: Conditions above imply with $\gamma\colon [a, b[\ \cap C \to {}^{(\omega)}\mathbb{K}$ that

$$F(\gamma(b)) - F(\gamma(a)) = \uparrow_\gamma {}^1F(\zeta)\downarrow\zeta.$$

Proof:
$$\begin{aligned}F(\gamma(b)) - F(\gamma(a)) &= +_{s\in G} F(\vec{\gamma}(s)) - F(\gamma(s)) = +_{s\in G} {}^1F(\gamma(s))(\vec{\gamma}(s) - \gamma(ts))\\ &= {}^1F(\gamma(s))\,{}^1\gamma(s)\downarrow \quad = \quad \uparrow_\gamma {}^1F(\zeta)\downarrow\zeta.\end{aligned}$$

Corollary: If f has an AD F on a closed path (CP) γ, it holds with the conditions above that $\uparrow_\gamma f(\zeta)\downarrow\zeta = 0$. □

Integral formula: The last corollary shows that for $f\colon A \to {}^{(\omega)}\mathbb{C}$ and the CP $\gamma([a, b[) \subseteq A \to {}^{(\omega)}\mathbb{C}$ the equation (see below) $f(z)\,\mathrm{ind}_\gamma(z) = \underline{\tilde{\tilde{\pi}}} \uparrow_\gamma \widetilde{\zeta - z} f(\zeta) \downarrow \zeta$ holds, if and only if $g(\zeta) = \widetilde{\zeta - z}(f(\zeta) - f(z))$ implies that $\uparrow_\gamma g(\zeta) \downarrow \zeta = 0$, This is especially true if g has an AD on $\gamma([a, b[)$.□

Remark: The conventionally real case of both fundamental theorems may be established analogously. Given $u, v \in G, u \neq v$ and $\gamma(u) = \gamma(v)$, it may be the case that $\vec{\gamma}(u) \neq \vec{\gamma}(v)$.

Remark: In the first fundamental theorem, the derivative $\downarrow F(z)/\downarrow z$ can be tightened to the arithmetic mean $\tilde{2}(f(z) + f(\vec{z}))$ resp. $f(\tilde{2}(z + \vec{z}))$, and similarly, in the second fundamental theorem, $F(\gamma(b)) - F(\gamma(a))$ can be tightened to $\tilde{2}(F(\gamma(b)) + F(\vec{\gamma}(b))) - \tilde{2}(F(\gamma(a)) + F(\vec{\gamma}(a)))$ resp. $F(\tilde{2}(\gamma(b) + \vec{\gamma}(b))) - F(\tilde{2}(\gamma(a) + \vec{\gamma}(a)))$. This yields approximately the original results when f and F are sufficiently α-continuous at the boundary.

Example: The alternating harmonic series implies $\pm_{n=1}^{\omega}(\tilde{n} - \omega) = {}_\epsilon 2$.

Leibniz integral rule: For $f\colon {}^{(\omega)}\mathbb{K}^{\grave{n}} \to {}^{(\omega)}\mathbb{K}$, $a, b\colon {}^{(\omega)}\mathbb{K}^n \to {}^{(\omega)}\mathbb{K}$, $\vec{x} := (s, x_2, \ldots, x_n)^{\mathrm{T}}$ and $s \in {}^{(\omega)}\mathbb{K} \setminus \{x_1\}$ choosing $\vec{a}(x) = a(\vec{x})$ and $\vec{e}(x) = e(\vec{x})$, it holds that

$$\frac{\downarrow}{\downarrow x_1}\left(\uparrow_{a(x)}^{e(x)} f(x,t)\downarrow t\right) = \left(\uparrow_{a(x)}^{e(x)} \frac{\downarrow f(x,t)}{\downarrow x_1}\downarrow t\right) + \frac{\downarrow e(x)}{\downarrow x_1} f(\vec{x}, e(x)) - \frac{\downarrow a(x)}{\downarrow x_1} f(\vec{x}, a(x)).$$

Proof:

$$\frac{\downarrow}{\downarrow x_1}\left(\uparrow_{a(x)}^{e(x)} f(x,t)\downarrow t\right) = \left(\uparrow_{a(\vec{x})}^{e(\vec{x})} f(\vec{x},t)\downarrow t - \uparrow_{a(x)}^{e(x)} f(x,t)\downarrow t\right)/\downarrow x_1$$
$$= \left(\left(\uparrow_{a(x)}^{e(x)}(f(\vec{x},t) - f(x,t))\downarrow t\right) + \uparrow_{b(x)}^{e(\vec{x})} f(\vec{x},t)\downarrow t - \uparrow_{a(x)}^{a(\vec{x})} f(\vec{x},t)\downarrow t\right)/\downarrow x_1$$
$$= \left(\uparrow_{a(x)}^{e(x)} \frac{\downarrow f(x,t)}{\downarrow x_1}\downarrow t\right) + \frac{\downarrow e(x)}{\downarrow x_1} f(\vec{x}, e(x)) - \frac{\downarrow a(x)}{\downarrow x_1} f(\vec{x}, a(x)). \square$$

Remark: Complex integration allows a path whose start and end points are the limits of integration. If $\vec{a}(x) \neq a(\vec{x})$, then multiply the final summand by $(\vec{a}(x) - a(x))/(a(\vec{x}) - a(x))$, and if $\vec{e}(x) \neq e(\vec{x})$, then multiply the penultimate summand by $(\vec{e}(x) - e(x))/(e(\vec{x}) - e(x))$. Let $n \in {}^{\omega}\mathbb{N}^*$ and $x \in [0, 1]$ in each case for the following examples (cf. [813], p. 540 – 543).

1. The sequence $f_n(x) = \sin nx/n^{\tilde{2}}$ does not tend to $f(x) = 0$ as $n \to \omega$, but instead to $f(x) = \tilde{\omega}^{\tilde{2}} \sin \omega x$ with (continuous) derivative ${}^1f(x) = \omega^{\tilde{2}} \cos \omega x$ instead of ${}^1f(x) = 0$.

2. The sequence $f_n(x) = x - \tilde{n}x^n$ tends to $f(x) = x - \tilde{\omega}x^\omega$ as $n \to \omega$ instead of $f(x) = x$ with (continuous) derivative ${}^1f(x) = 1 - x^{\acute{\omega}}$ instead of ${}^1f(x) = 1$. Conventionally, $f_n(x) = 1 - x^{\acute{n}}$ is discontinuous at $x = 1$.

3. The sequence $f_n(x) = nx(-\acute{x})^n$ does not tend to $f(x) = 0$ as $n \to \omega$, but to the continuous function $f(x) = \omega x(-\acute{x})^\omega$, and takes the value $\tilde{\epsilon}$ when $x = \tilde{\omega}$.

Definition: The direction $w := \vec{z}$ gives the *second derivative* of $f\colon A \to {}^{(\omega)}\mathbb{K}$ at $z \in A \subseteq {}^{(\omega)}\mathbb{K}$ by

$${}^2f(\vec{z}) := \frac{\downarrow^2 f(\vec{z})}{(\downarrow \vec{z})^2} := \frac{f(\vec{w}) - \hat{f}(\vec{z}) + f(z)}{(\downarrow \vec{z})^2}. \triangle$$

Remark: Higher derivatives are defined analogously. Every number $m_n \in {}^{\omega}\mathbb{N}$ for $n \in {}^{\omega}\mathbb{N}^*$ of derivatives is written as an exponent after the n-th variable to be differentiated. The exponent to be specified in the numerator is the sum of all m_n. Regarding $1/(-1)! = 0$ implies then for g like f and $p \in {}^{\omega}\mathbb{N}^*$ the Leibniz product rule:

$${}^p(fg) = +_{m+n=p} \binom{p}{m} {}^m f\, {}^n g.$$

Proof: For $p = 1$, the product rule mentioned above holds. Induction step from p to $\grave{p}$:

$${}^{\grave{p}}(fg) \underset{p}{=} +_{m+1+n=\grave{p}} \left(\binom{p}{m} + \binom{p}{\grave{m}}\right) {}^{\grave{m}} f\, {}^n g + +_{m+1+n=\grave{p}} \binom{p}{m} {}^m f\, {}^{\grave{n}} g - +_{m+1+n=\grave{p}} \binom{p}{\grave{m}} {}^{\grave{m}} f\, {}^n g$$

$${}^p({}^1(fg)) \underset{1}{=} {}^p({}^1f g + f\, {}^1g) = {}^p({}^1fg) + {}^p(f\, {}^1g) = +_{m+n=\grave{p}} \binom{\grave{p}}{m} {}^m f\, {}^n g. \square$$

Taylor's theorem: $+_m |\,^m f(a)| > \tilde{\mathrm{v}}$, $^m f(a) \in {}^\omega\mathbb{C}$, $g(z) := (z-a)^\omega$, $|z-a| < \tilde{\epsilon}\omega$ and $z \to a \in {}^\omega\mathbb{C}$ imply

$$f(z) = T_\omega(z) := +_{m=0}^{\omega} \widetilde{m!}\ {}^m f(a)(z-a)^m.$$

Proof: From L'Hôpital's rule, it follows respecting the Leibniz product rule that

$$f(z) = \frac{(fg)(z)}{g(z)} = \frac{{}^1(fg)(z)}{{}^1g(z)} = \cdots = \frac{{}^{\acute{\omega}}(fg)(z)}{{}^{\acute{\omega}}g(z)} = \frac{{}^\omega(fg)(z)}{{}^\omega g(z)} = \widetilde{\omega!}\ {}^\omega(fg)(z)$$

and

$${}^\omega(fg)(z) = +_{m+n=\omega} \binom{\omega}{m}\ {}^m f(a)\ {}^{\omega-m}g(z) = {}^\omega g(z) +_{m=0}^{\omega} \widetilde{m!}\ {}^m f(a)(z-a)^m\ .\square$$

Conclusion: The second fundamental theorem implies for the remainder $R_n(z) := f(z) - T_n(z) = f(a) + \uparrow_a^z\ {}^1f(t) \downarrow t - T_n(z)$ by the mean value theorem where $\zeta \in {}^a\dot{\mathbb{C}}(z)$ and $p \in \mathbb{N}^*_{\le n}$

$$R_n(z) = \uparrow_a^z \widetilde{n!}\,(z-t)^n\ {}^{\grave{n}}f(t)\downarrow t = \widetilde{pn!}\,(z-\zeta)^{\grave{n}-p}\ {}^{\grave{n}}f(\zeta)(z-a)^p.$$

Proof by induction with integration by parts and induction step from $\acute{n}$ to n ($\acute{n} = 0$ see above):

$$f(z) = T_{\acute{n}}(z) + \widetilde{n!}\,(z-a)^n\ {}^n f(a) + \uparrow_a^z \widetilde{n!}\,(z-t)^n\ {}^{\grave{n}}f(t)\downarrow t = T_n(z) + R_n(z).\square$$

Remark: It holds that $(\epsilon^\iota - 1)/\iota = 1 = {}^1\exp(0)$ and thus $\downarrow_\epsilon y/\downarrow y = \tilde{y}$ from $\downarrow y/\downarrow x = y := \epsilon^x$ as well as $\downarrow x^n = \downarrow \epsilon^{n\,\epsilon^x} = nx^{\acute{n}}\downarrow x$ for $n \in {}^\omega\mathbb{N}^*$ by product and chain rule. Unit circle and triangles easily show the relations $\sin\iota/1 = (\cos\iota - 1)/\iota$ and $\cos\iota/1 = -\sin\iota/\iota$. Hence, it holds ${}^1\sin(0) = \cos(0)$ and ${}^1\cos(0) = -\sin(0)$ as well as for $m \in {}^\omega\mathbb{N}$ and $n = \hat{k}$ de Moivre's formula:

$$\left(\cos(z) + \underline{\sin}(z)\right)^m = \epsilon^{\underline{m}z} = 1 + +_{k=1}^{\breve{\omega}}\left(\widetilde{\acute{n}!}\,(\underline{m}z)^{\acute{n}} + \widetilde{n!}\,(\underline{m}z)^n\right) = \cos(mz) + \underline{\sin}(mz).\square$$

Euler's sine formula: Zero and identity theorem (cf. [472], p. 41) for series plus the theorem above analogously yield $\Gamma(\tilde{2}) = \pi^{\tilde{2}}$ for the gamma function $\Gamma(z) := \omega!\,\omega^z / \times_{k=0}^{\omega}(z+k)$ where $z \in {}^\mathrm{v}\mathbb{C} \setminus -{}^\mathrm{v}\mathbb{N}$ from

$$\frac{\epsilon^{\underline{\hat{\pi}}z} - 1}{\epsilon^{\underline{\pi}z}\underline{\hat{\pi}}z} = \frac{\epsilon^{\underline{\pi}z} - \epsilon^{-\underline{\pi}z}}{\underline{\hat{\pi}}z} = \frac{\sin(\pi z)}{\pi z} = +_{k=0}^{\omega} \frac{(\underline{\pi}z)^n}{\grave{n}!} = \times_{k=1}^{\omega}(1 - z^2/k^2) = \frac{\tilde{z}}{\Gamma(z)\Gamma(-\acute{z})},$$

since all $\hat{\omega}$ zeros of the left- and right-hand side match due to $\epsilon^{\underline{\pi}n} = 1 + \sin 0$.$\square$

Conclusion: This shows for the Wallis product $W := +_{k=1}^{\omega} k^2/(k^2 - \tilde{4}) = \breve{\pi}$ from

$$\frac{\Gamma(\tilde{2})^2}{\widehat{W}} = \frac{\breve{\omega}4^{\grave{\omega}}\omega!^2}{(\hat{\omega}+1)!!^2}\frac{(\hat{\omega}+1)(\hat{\omega}-1)!!^2}{4^\omega\omega!^2} = \frac{\hat{\omega}}{\hat{\omega}+1} := 1 = \frac{\breve{\pi}}{W}.\square$$

Functional equation of the Gamma function: From $\Gamma(\grave{z}) = z\Gamma(z)\omega/(\omega + \grave{z})$ and $\Gamma(1) := 1$, it follows for sufficiently small $|z|$ integrating by parts that $\Gamma(\grave{z}) := \uparrow_0^\omega t^z\epsilon^{-t}\downarrow t = z\Gamma(z)$. This leads for $z = \tilde{2}$ and the substitution $x := t^{\tilde{2}}$ to the equation $\uparrow_0^\omega \epsilon^{-x^2}\downarrow x = \tilde{2}\pi^{\tilde{2}}$ relevant to statistics and ball computation.$\square$

Conclusion: A logarithmic derivative (see [473], p. 324) shows the equation $\uparrow_0^\omega \tilde{x}\sin x \downarrow x = \breve{\pi}$.$\square$

Stirling formula: Putting $\theta = 12\omega$, the asymptotic approximation $\grave{\theta}^{-1} < {}_\epsilon\left(\omega!/\,(\hat{\pi}\omega)^{\tilde{2}}(\tilde{\epsilon}\omega)^\omega\right) < \tilde{\theta}$.

Proof: Logarithmise $\binom{\hat{\omega}}{\omega} = \frac{(\tilde{\pi}\omega)^{\tilde{2}}4^\omega}{\omega+\tilde{2}} \sim \frac{4^\omega}{(\pi\omega)^{\tilde{2}}}$ as before to obtain $\omega! = d(\pi\omega)^{\tilde{2}}(c\omega)^\omega$ for $c, d \in {}^\mathrm{v}\mathbb{R}_{>0}$ from

$$+_{n=1}^{\omega}\ {}_\epsilon n + {}_\epsilon 4\ \omega - \tilde{2}_\epsilon(\pi\omega) = +_{n=\grave{\omega}}^{\hat{\omega}}\ {}_\epsilon n = \omega_\epsilon(b\omega),$$

where $b \in]1, 2[$. Subscripting $c_{\grave{\omega}}/c_\omega$ of c yields $c = \tilde{\epsilon}$ and $d_{\hat{\omega}}^2/d_{\hat{\omega}}$ of d reveals $d = 2^{\tilde{2}}$. TS (see above) and GS of the logarithm show $\theta = 12\omega$ from $\acute{\upsilon} = \hat{\omega}$ and $\tilde{\theta} - \grave{\theta}^{-1} + 1 = (\omega + \tilde{2})_\epsilon(\tilde{\omega}\grave{\omega}) = \breve{\upsilon}((1+\tilde{\upsilon})/(1-\tilde{\upsilon}))$.$\square$

Factorial theorem: For $n \in {}^{\omega}\mathbb{N}$, the Stirling formula $n! \sim (\hat{\pi}n)^{\tilde{2}}(\tilde{e}n)^n \epsilon^{\widetilde{12n}}$ yields asymptotically $n! \in [g(s_+), g(s_-)]$ if $\tilde{s}_\pm = \tilde{2}(25 \pm 1)n^{\operatorname{sgn}(\acute{n})}$ and $g(s_\pm) = \left[\left(\hat{\pi}n + \tilde{3}\grave{s}_\pm \pi\right)^{\tilde{2}}(\tilde{e}n)^n\right]$ holds .

Proof: Use $\tilde{t} := 6n$ for $t = {}_\epsilon(1 + \grave{s}_\pm t) = (\grave{s}_\pm - \check{t})t + \mathcal{O}(t^3)$ by mathematical induction.□

Green's theorem: Given NRs $B \subseteq \mathbb{D}^2$ for some h-domain $\mathbb{D} \subseteq {}^{(\omega)}\mathbb{R}^2$,), sufficiently large $m \in \mathbb{N}^*$, infinitesimal $h = |\downarrow x| = |\downarrow y| = |\vec{\gamma}(t) - \gamma(t)| = \mathcal{O}(\widetilde{\omega}^m$ $(x, y) \in \mathbb{D}$, $\mathbb{D}^- := \{(x, y) \in \mathbb{D} : (x + h, y + h) \in \mathbb{D}\}$, and a simply CP γ: $[a, b[\to \partial\mathbb{D}$ followed anticlockwise, choosing $\vec{\gamma}(s) = \gamma(\vec{s})$ for $s \in [a, b[$, $A \subseteq [a, b]^2$, the following equation holds for sufficiently α-continuous functions u, v: $\mathbb{D} \to \mathbb{R}$ with not necessarily continuous $\downarrow u/\downarrow x$, $\downarrow u/\downarrow y$, $\downarrow v/\downarrow x$ and $\downarrow v/\downarrow y$

$$\uparrow_\gamma (u\downarrow x + v\downarrow y) = \uparrow_{(x,y)\in\mathbb{D}^-} \left(\frac{\downarrow v}{\downarrow x} - \frac{\downarrow u}{\downarrow y}\right)\downarrow(x, y).$$

Proof: Only $\mathbb{D} := \{(x, y) : r \le x \le s, f(x) \le y \le g(x)\}$, $r, s \in {}^{(\omega)}\mathbb{R}$, $f, g : \partial\mathbb{D} \to {}^{(\omega)}\mathbb{R}$ is proved, since the proof is analogous for each case rotated by ι. Every h-domain is union of such sets. Simply showing

$$\uparrow_\gamma u\downarrow x = -\uparrow_{(x,y)\in\mathbb{D}^-} \frac{\downarrow u}{\downarrow y}\downarrow(x,y)$$

is sufficient because the other relation is given analogously. Neglecting the regions of γ with $\downarrow x = 0$ and $s := h(u(r, g(r)) - u(t, g(t)))$ shows

$$\uparrow_\gamma u\downarrow x - s = \uparrow_t^r u\big(x, g(x)\big)\downarrow x - \uparrow_t^r u\big(x, f(x)\big)\downarrow x = \uparrow_t^r \uparrow_{f(x)}^{g(x)} \frac{\downarrow u}{\downarrow y}\downarrow y\downarrow x = -\uparrow_{(x,y)\in\mathbb{D}^-} \frac{\downarrow u}{\downarrow y}\downarrow(x,y)\,. \square$$

Remarks: If the moduli of $x \in \mathbb{C}$, $\downarrow x$ or $\widetilde{\downarrow x}$ have different orders of magnitude, the identity

$${}^0s(x) := \pm_{m=0}^n x^m = \left(1 - x -^{\grave{n}}\right)/\grave{x}$$

yields by differentiating

$${}^1s(x) := \mp_{m=1}^n m x^{\acute{m}} = \left(\grave{n}x -^n - nx -^{\grave{n}} - 1\right)/\grave{x}^2.$$

The formulas above were sometimes miscalculated. For sufficiently small x, and sufficiently, but not excessively large n, the latter can be further simplified to $-\grave{x}^{-2}$ and remains valid when $x \ge 1$ is not excessively large. By successively multiplying ${}^m s(x)$ by x for $m \in {}^{\omega}\mathbb{N}^*$ and subsequently differentiating, other formulas can be derived for ${}^{\grave{m}}s(x)$, providing an example of divergent series. However, if ${}^0s(-x)$ is integrated from 0 to 1 putting $n := \omega$, an integral expression for ${}_\epsilon\omega + \gamma$ is obtained for Euler's constant γ.

L'Hôpital's rule solves the case of $x = -1$. Substituting $y := -\acute{x}$, by the binomial series a series is obtained with infinite coefficients (if ${}_\epsilon\omega$ is also expressed as a series, even an expression for γ is obtained). If the numerator of ${}^0s(x)$ is illegitimately simplified, finding incorrect results is risked, especially when $|x| \ge 1$. So ${}^0s(-\epsilon^{\underline{\pi}})$ is e.g. 0 for odd n, and 1 for even n, but not $\tilde{2}$.

Theorem: Splitting F: $A \to {}^{(\omega)}\mathbb{C}$ into real and imaginary parts $F(z) := U(z) + \underline{V}(z) := f(x, y) := u(x, y) + \underline{v}(x, y)$, h-homogeneous $A \subseteq {}^{(\omega)}\mathbb{C}$ and $h = |\downarrow x| = |\downarrow y|$, with the NR $B \subseteq A^2$ for every $z = x + \underline{y} \in A$, F is holomorphic if and only if the *Cauchy-Riemann differential equations* are satisfied:

$$\frac{\downarrow u}{\downarrow x} = \frac{\downarrow v}{\downarrow y}, \frac{\downarrow v}{\downarrow x} = -\frac{\downarrow u}{\downarrow y}.$$

Proof: The claim follows directly from $\frac{\downarrow u}{\downarrow x} + \frac{\downarrow \underline{v}}{\downarrow x} = \frac{\downarrow v}{\downarrow y} - \frac{\downarrow \underline{u}}{\downarrow y} = \frac{\downarrow F}{\downarrow z} = \downarrow U(z) + \downarrow \underline{V}(z)$.□

Remark: The following necessary and sufficient condition is valid for F to be holomorphic:

$$F'(\bar{z}) = \frac{\downarrow f}{\downarrow x} = \frac{\downarrow \underline{f}}{\downarrow y} = \tilde{2}\left(\frac{\downarrow f}{\downarrow x} + \frac{\downarrow \underline{f}}{\downarrow y}\right) = \frac{\downarrow F}{\downarrow \bar{z}} = 0.$$

Goursat's integral lemma: If $f \in \mathcal{O}(\Delta)$ on a triangle $\Delta \subseteq {}^{(\omega)}\mathbb{C}$ but has no AD on Δ, then (cf. [473], S. 149 ff.)

$$I := \uparrow_{\partial\Delta} f(\zeta)\downarrow\zeta = 0.$$

Refutation of conventional proofs based on estimation by means of a complete triangulation: The direction in which $\partial\Delta$ is traversed is irrelevant. If Δ is fully triangulated, then wlog every minimal triangle $\Delta_s \subseteq \Delta$ must either satisfy where κ, λ and μ represent the vertices of Δ_s

$$I_s := \uparrow_{\partial\Delta_s} f(\zeta)\downarrow\zeta = f(\kappa)(\lambda-\kappa) + f(\lambda)(\mu-\lambda) + f(\kappa)(\kappa-\mu) = \big(f(\kappa)-f(\lambda)\big)(\lambda-\mu) = 0$$

or

$$\begin{aligned}\uparrow_{\partial\Delta_s} f(\zeta)\downarrow\zeta &= f(\kappa)(\lambda-\kappa) + f(\lambda)(\mu-\lambda) + f(\mu)(\kappa-\mu) = \big(f(\kappa)-f(\lambda)\big)\lambda + \big(f(\lambda)-f(\mu)\big)\mu + \big(f(\mu)-f(\kappa)\big)\kappa \\ &= {}^1f(\lambda)\big((\kappa-\lambda)\lambda + (\mu-\lambda)\mu + (\mu-\lambda)\kappa + (\kappa-\lambda)\kappa\big) = {}^1f(\lambda)\big((\mu-\lambda)(\kappa-\mu) - (\kappa-\lambda)^2\big) = 0.\end{aligned}$$

By holomorphicity and cyclic permutations, this can only happen for $f(\kappa) = f(\lambda) = f(\mu)$.

If every adjacent triangle to Δ is considered, deduce that f must be constant, which contradicts the assumptions. This is because the term in large brackets is translation-invariant, since otherwise put $\mu := 0$ wlog, making this term 0, in which case $\kappa = \check{\lambda}\big(1 \pm 3 - \tilde{2}\big)$ and $|\kappa| = |\lambda| = |\kappa - \lambda|$. However, since every horizontal and vertical line is homogeneous on ${}^{(\omega)}\mathbb{C}$, this cannot happen:

Otherwise, the corresponding sub-triangle would be equilateral and not isosceles and right-angled. Therefore, in both cases, $|I_s|$ must be at least $|{}^1f(\lambda)\ \mathcal{O}(\iota^2)|$, by selecting the vertices 0, ι and $\underline{\iota}$ wlog. If L is the perimeter of a triangle, then it holds that $|I| \leq 4^m\ |I_s|$ for an infinite natural number m, and also $2^m = (\partial\Delta)/|(\iota^2)|$ since $L(\partial\Delta) = 2^m\ L(\partial\Delta_s)$ and $L(\partial\Delta_s) = |\mathcal{O}(\iota^2)|$. It holds that $|I| \leq |{}^1f(\lambda)\ L(\partial\Delta)^2/\mathcal{O}(\iota^2)|$, causing the desired estimate $|I| \leq |\mathcal{O}(\downarrow\zeta)|$ to fail, for example if $|{}^1f(\lambda)\ L(\partial\Delta)^2|$ is larger than $|\mathcal{O}(\iota^2)|$.□

Cauchy's integral theorem: Given the NRs $B \subseteq \mathbb{D}^2$ and $A \subseteq [a, b]$ for some h-domain $\mathbb{D} \subseteq {}^\omega\mathbb{C}$, infinitesimal h, $f \in \mathcal{O}(\mathbb{D})$ and a CP γ: $[a, b[\to \partial\mathbb{D}$, choosing $\vec{\gamma}(s) = \gamma(\vec{s})$ for $s \in [a, b[$ gives $\uparrow_\gamma f(z)\downarrow z = 0$.

Proof: By the Cauchy-Riemann differential equations and Green's theorem, with $x := \operatorname{Re} z$, $y := \operatorname{Im} z$, $u := \operatorname{Re} f$, $v := \operatorname{Im} f$, and $\mathbb{D}^- := \{z \in \mathbb{D} : z + h + \underline{h} \in \mathbb{D}\}$, it holds that

$$\uparrow_\gamma f(z)\downarrow z = \uparrow_\gamma\big(u + \underline{v}\big)\big(\downarrow x + \downarrow\underline{y}\big) = \uparrow_{z\in\mathbb{D}^-}\left(\left(\frac{\downarrow\underline{u}}{\downarrow x} - \frac{\downarrow\underline{v}}{\downarrow\underline{y}}\right) - \left(\frac{\downarrow v}{\downarrow x} + \frac{\downarrow u}{\downarrow y}\right)\right)\downarrow(x, y) = 0.\ \square$$

Remark: For $\widetilde{\omega} := 0$, the main theorem of Cauchy's theory of functions can be proven according to Dixon (as in [473], p. 228 f.), since the limit there shall be 0 resp. $\tilde{r}$ tends to 0 for $r \in {}^\omega\mathbb{R}_{>0}$ tending to ω. The in ${}^\omega\dot{\mathbb{C}} \subset {}^\omega\mathbb{C}$ (entire) functions $f(z) = \bigplus_{k=1}^{\omega} z^k\widetilde{\omega}^{k+1}$ and $g(z) = \widetilde{\omega}z$ give counterexamples to Liouville's (generalised) theorem and Picard's little theorem because of $|f(z)| \leq 1$ and $|g(z)| \leq 1$. The function $f(\tilde{z})$ for $z \in {}^\omega\dot{\mathbb{C}}^*$ discounts Picard's great theorem.

Counter-directional theorem: If the path γ: $G = [a, b[\cap C \to V$ with $C \subseteq \mathbb{R}$ passes the edges of every n-cube of side length ι in the n-volume $V \subseteq {}^{(\omega)}\mathbb{R}^n$ with $n \in \mathbb{N}_{\geq 2}$ exactly once, where the opposite edges in all two-dimensional faces of every n-cube are traversed in reverse direction, but uniformly, then, for $D \subseteq \mathbb{R}^2$, $B \subseteq V^2$, $f = (f_1, ..., f_n)$: $V \to {}^{(\omega)}\mathbb{R}^n$, $\gamma(s) = x$, $\gamma(\vec{s}) = \vec{x}$ and $V_r := \{\vec{x} \in V: x \in V, \vec{x} \neq \tilde{x}\}$, it holds that

$$\uparrow_{s\in G} f\big(\gamma(s)\big)\ {}^1\gamma(s)\downarrow s = \uparrow_{\substack{(x,\vec{x})\\ \in V\times V_r}} f(x)\downarrow Bx = \uparrow_{\substack{s\in G,\\ \gamma|\partial^{\acute{n}}V}} f\big(\gamma(s)\big)\ {}^1\gamma(s)\downarrow s.$$

Proof: If two arbitrary squares are considered with common edge of length ι included in one plane, then only the edges of $V\times V_r$ are not passed in both directions for the same function value. They all, and thus the path to be passed, are exactly contained in $\partial^{\acute{n}}V$.□

Transformation theorem: If the Jacobian $D\varphi(x)$ exists, linear algebra shows for $f : \varphi(A) \to {}^{(\omega)}\mathbb{R}^n$ and $A \subseteq {}^\omega\mathbb{R}^n$ (cf. [764], S. 519):

$$\uparrow_{\varphi(A)} f(y)\downarrow y = \uparrow_A f(\varphi(x))|\operatorname{eig}(D\varphi(x))|\downarrow x.\ \square$$

Definition: For a CP γ: $[a, b[\to {}^{(\omega)}\mathbb{C}$ and $z \in {}^{(\omega)}\mathbb{C}$, $\tilde{\underline{\hat{n}}}\uparrow_\gamma \widetilde{\zeta - z}\downarrow\zeta$ is called *winding number* or *index* $\operatorname{ind}_\gamma(z) \in \mathbb{Z}$. The coefficients $a_{j,-1}$ of the function f: $A \to {}^{(\omega)}\mathbb{C}$ for $A \subseteq {}^{(\omega)}\mathbb{C}$, $n \in {}^\omega\mathbb{N}$, a_{jk}, $c_j \in {}^{(\omega)}\mathbb{C}$ and

$$f(z) = +_{j=0}^{n} +_{k=-\omega}^{\omega} a_{jk}(z-c_j)^k$$

as well as pairwise different c_j are called *residues* $\mathrm{res}_{c_j} f$.△

Residue theorem: For γ and f as above, it holds that $\underline{\tilde{\tilde{\pi}}} \uparrow_\gamma f(\zeta) \downarrow \zeta = +_{j=0}^{n} \mathrm{ind}_\gamma(c_j) \mathrm{res}_{c_j} f$.

Proof: For all $k \in {}^{\omega}\mathbb{Z} \setminus \{-1\}$, it holds $+_{j=0}^{n} \left| \uparrow_\gamma a_{jk} \left(\zeta - c_j\right)^k \downarrow \zeta \right| = 0$ and $\underline{\tilde{\tilde{\pi}}} \uparrow_\gamma a_{j,-1} \widetilde{\zeta - c_j} \downarrow \zeta = \mathrm{ind}_\gamma(c_j) \mathrm{res}_{c_j} f$.□

Definition: A point $z_0 \in M \subseteq {}^{(\omega)}\mathbb{C}^n$ or of a sequence (a_k) for $k \in {}^{(\omega)}\mathbb{N}$ is called a *(proper) α-accumulation point* of M or of the sequence if the ball ${}^{\alpha}\dot{\mathbb{C}}^n(z_0) \subseteq {}^{(\omega)}\mathbb{C}^n$ with centre z_0 and infinitesimal α contains infinitely many points from M or pairwise distinct members of $a_k \in {}^{(\omega)}\mathbb{C}^n$. Let α- be omitted for $\alpha = \tilde{\omega}$.△

Remark: Choose the pairwise distinct zeros $c_k \in {}^{\tilde{\omega}}\dot{\mathbb{C}} \subset \mathbb{D}$ for $z \in {}^{\omega}\mathbb{C}$ in $p(z) = \times_{k=0}^{\omega}(z - c_k)$ in such a way that $|f(c_k)| < \tilde{\omega}$ for $f \in \mathcal{O}(\mathbb{D})$ on a domain $\mathbb{D} \subseteq \mathbb{C}$ where $f(0) = 0$. Let $\mathbb{D}$ contain ${}^{\tilde{\omega}}\dot{\mathbb{C}}$ completely, which a coordinate transformation always achieves provided that $\mathbb{D}$ is sufficiently "large". The coincidence set $\{\zeta \in \mathbb{D} : f(\zeta) = g(\zeta)\}$ of $g(z) := f(z) + p(z) \in \mathcal{O}(\mathbb{D})$ contains an accumulation point at 0. Since $p(z)$ can take every conventional complex number, the deviation between f and g is non-negligible.

Since $f \neq g$, this contradicts the statement of the identity theorem like the (local) fact that all derivatives ${}^{n}u(z_0) = {}^{n}v(z_0)$ of two functions u and v can be equal at $z_0 \in \mathbb{D}$ for all n, but u and v may significantly differ further away maintaining to be holomorphic, since some holomorphic function has to be developed into a TS with approximated powers. The function $b(z) := \tilde{\nu}z$ for $z \in {}^{\nu}\dot{\mathbb{C}} \subset {}^{\nu}\mathbb{C}$ maps the simply connected ${}^{\nu}\dot{\mathbb{C}}$ holomorphicly to ${}^{1}\dot{\mathbb{C}}$.

A missing injectivity or surjectivity requires correcting the Riemann mapping theorem. Examples of such $f \in \mathcal{O}(\mathbb{D})$ include functions with $f(0) = 0$ that are restricted to ${}^{\tilde{\omega}}\dot{\mathbb{C}}$. Extending the upper limit from ω to $|\mathbb{N}^*|$ gives entire functions with an infinite number of zeros. The set of zeros is not necessarily discrete. Thus, the set of all functions $f \in \mathcal{O}(\mathbb{D})$ may contain zero divisors. The functions once again give counterexamples to Picard's little theorem since they omit at least $\acute{n}$ values in ${}^{\omega}\mathbb{C}$.

Theorem (binomial series): From $\alpha \in {}^{\nu}\mathbb{C}$, $\binom{\alpha}{n} := \tilde{n}!\, \alpha\acute{\alpha} \ldots (\grave{\alpha} - n)$ and $\left|\binom{\alpha}{\grave{m}} / \binom{\alpha}{m}\right| < 1$ for all $m \geq \nu$ where $\binom{\alpha}{0} := 1$, it follows for $z \in \mathbb{D}^{\ll}$ or $z \in {}^{(\omega)}\mathbb{C}$ für $\alpha \in {}^{(\omega)}\mathbb{N}$ the TS centred on 0 that $\grave{z}^{\alpha} = +_{n=0}^{\omega} \binom{\alpha}{n} z^n$.□

Multinomial theorem: For $\zeta \in {}^{\omega}\mathbb{C}$, $z \in {}^{\omega}\mathbb{C}^k$, $k \in {}^{\omega}\mathbb{N}_{\geq 2}$, $m, n_j \in {}^{\omega}\mathbb{N}^*$, $|n| := +_{j=1}^{k} n_j$, $z^n := \times_{j=1}^{k} z_j^{n_j}$ and $\binom{m}{n} := \widetilde{n_1! \ldots n_k!}\, m!$, it holds that

$$(1\ 1_k^T\ z)^m = +_{|n|=m} \binom{m}{n} z^n .$$

Proof: Cases $k \in \{1, 2\}$ are clear. Induction step from k to $\grave{k}$ where $\binom{m}{n} = \binom{m}{n_1, \ldots, n_k, p} \binom{p}{n_k, n_{\grave{k}}}$ and $p = n_k + n_{\grave{k}}$:

$$\left(1\ 1_{\grave{k}}^T\ z\right)^m \Big|_{\zeta_k = z_k + z_{\grave{k}}} = +_{|n|=m} \binom{m}{n} z^n \Big|_{\eta_k! = n_k! n_{\grave{k}}!} = +_{|n|=m} \binom{m}{n} z^n .\square$$

resp. from m to $\grave{m}$

$$\begin{aligned}(1\ 1_k^T\ z)^{\grave{m}} &= \grave{m} \uparrow_0^{z_j} (1\ 1_k^T\ z)^m |_{z_j=\zeta} \downarrow \zeta + (1\ 1_k^T\ z)^{\grave{m}} \big|_{z_j=0} \\ &= \grave{m} \uparrow_0^{z_j} +_{|n|=m} \binom{m}{n} z^n |_{z_j=\zeta} \downarrow \zeta + (1\ 1_k^T\ z)^{\grave{m}} \big|_{z_j=0} = +_{|\grave{n}|=\grave{m}} \binom{\grave{m}}{\grave{n}} z^{\grave{n}}\end{aligned}$$

General Leibniz formula: Putting $\downarrow^n := \downarrow_1^{n_1} \ldots \downarrow_k^{n_k}$ and $\downarrow_j^{n_j} := \downarrow^{n_j} / \downarrow z_j^{n_j}$, it follows for $j, k, m, n \in {}^{(\omega)}\mathbb{N}$ and differentiable $f = f_1 \cdot \ldots \cdot f_k \in {}^{(\omega)}\mathbb{C}$ from the multinomial theorem $\downarrow^m f = +_{|n|=m} \binom{m}{n} \downarrow^n f$.□

Example: Putting $f(x) := +_{n=1}^{\omega} \tilde{n}^2\, {}_{\epsilon}(1 + n^2x^2)$ implies ${}^{1}f(0) = \frac{f(\iota) - f(0)}{\iota - 0} = +_{n=1}^{\omega} \frac{x^2}{1 + n^2x^2}\Big|_0 = +_{n=1}^{\omega} \tilde{\iota}\tilde{n}^2\, {}_{\epsilon}(1 + n^2\iota^2) = \iota\omega = 0$, where the series expansion ${}_{\epsilon}\grave{x} = \pm_{n=1}^{\omega} \tilde{n}x^n$ for $x \in]-1,1[$ was used differentiating term by term.

Taylor's theorem for several variables: For $n! := \bigtimes_{j=1}^{k} n_j!$, $a, z \in {}^{(\omega)}\mathbb{C}^k$ and $(z-a)^n := \bigtimes_{j=1}^{k} (z-a)^{n_j}$, it follows from the multinomial theorem also analogously to the proof of the simple TS for $n \in {}^{(\omega)}\mathbb{N}$

$$f(z) = T_\omega(z) := \mathop{+}_{|n|=0}^{\omega}\left(\widetilde{n}! \downarrow^n f(a)(z-a)^n\right). \square$$

Conclusion: Analogously to the simple TS, the remainder is for $\zeta \in {}^{\alpha}\dot{\mathbb{C}}(z)$ and $k \in \mathbb{N}^*_{\leq \grave{n}}$

$$R_n(z) = (z-\zeta)^k/(1-k/\grave{n}) \mathop{+}_{|m|=\grave{n}} \widetilde{m}! \downarrow^m f(\zeta)(z-a)^{m-k}. \square$$

Chain rule for several variables: For $z \in A \subseteq {}^{(\omega)}\mathbb{K}^k$, $g: A \to B \subseteq {}^{(\omega)}\mathbb{K}^m$, $f: B \to C \subseteq {}^{(\omega)}\mathbb{K}^n$, it holds for $k, m, n \in {}^{(\omega)}\mathbb{N}^*$:

$${}^1f\big(g(z)\big) = {}^1f\big(g(z)\big)\,{}^1g(z)$$

Proof: Taylor's theorem for several variables implies with bounded $\|r(z)\|$ and $\|s(g(z))\|$

$$g(\vec{z}) = g(z) + {}^1g(z) \downarrow z + r(z)\|\downarrow z\|^2 \text{ and } f\big(\vec{g}(z)\big) = f\big(g(z)\big) + {}^1f\big(g(z)\big) \downarrow g(z) + s\big(g(z)\big)\|\downarrow g(z)\|^2. \square$$

Newton's method: Demanding above $f(\vec{z}) = f(z) + {}^1f(z) \downarrow z = 0$ implies $z_{\grave{n}} := z_n - {}^1f(z_n)^{-1} f(z_n)$ if ${}^1f(z_n)^{-1}$ is invertible resulting in quadratic convergence close to a zero.$\square$

Finiteness criterion for series: Let $m, n, q, r \in \mathbb{N}$. Sum $S_r := \left|\mathop{+}_{q=0}^{r} s_q\right|$ for $s_q \in {}^{(\omega)}\mathbb{C}$ is finite, if and only if $0 \leq S_r := \left|\mathop{\pm}_{m=0}^{n} a_m\right| \leq a_0$ for a sequence (a_m) such that $a_{\grave{m}} < a_m \in {}^{\nu}\mathbb{R}_{\geq 0}$ holds, as far as summands can be arbitrarily sorted according to their signs and sizes, recombined or split into separate sums.$\square$

Example: Putting $f(x) := \mathop{+}_{n=1}^{\omega} \tilde{n}^2\, {}_\epsilon(1+n^2x^2)$ implies ${}^1f(0) = \frac{f(\iota)-f(0)}{\iota-0} = \mathop{+}_{n=1}^{\omega} \frac{x^2}{1+n^2x^2}\Big|_0 = \mathop{+}_{n=1}^{\omega} \iota\tilde{n}^2\, {}_\epsilon(1+n^2\iota^2) = \iota\omega = 0$, where the series expansion ${}_\epsilon\grave{x} = \mathop{\pm}_{n=1}^{\omega} \tilde{n}x^n$ for $x \in\,]-1,1[$ was used differentiating term by term.

Theorem (series product): For $a_m, b_n \in {}^{(\omega)}\mathbb{K}$, the next equation replaces the Cauchy product (see [472], p. 103):

$$\mathop{+}_{m=1}^{\omega} a_m \mathop{+}_{k=1}^{\omega} b_n = \mathop{+}_{m=1}^{\omega}\left(\mathop{+}_{n=1}^{m}(a_n b_{m-\acute{n}} + a_{\omega-\acute{n}} b_{\omega-m+n}) - a_m b_{\omega-\acute{m}}\right). \square$$

Example: The following series product has the finite value (cf. [763], p. 61 f.):

$$\left(\mathop{\pm}_{m=1}^{\omega} \widetilde{m}^{\tilde{2}}\right)^2 = \mathop{+}_{m=1}^{\omega}\left(\left(\frac{\tilde{m}}{\omega-\acute{m}}\right)^{\tilde{2}} - \left(\underline{1}^{\widehat{m}} \mathop{+}_{n=1}^{m}\left(\left(\frac{\tilde{n}}{m-\acute{n}}\right)^{\tilde{2}} + \left(\frac{\widetilde{\omega-\acute{n}}}{\omega-m+n}\right)^{\tilde{2}}\right)\right)\right) = 0{,}36590\ldots \ll \frac{\zeta(\tilde{2})^2}{3+8^{\tilde{2}}}.$$

Example: The signum function sgn yields the following series product (cf. [763], p. 62):

$$\mathop{+}_{m=0}^{\omega}\left(2^{m^{\operatorname{sgn}(m)}} \mathop{+}_{n=0}^{m} \operatorname{sgn}(n-\gamma)\right) = \acute{\omega}2^{\grave{\omega}} \gg -2.$$

Stokes' theorem (cf. [764], p. 625 f.): If $^-$ stands for sufficiently α-continuous functions $f_m: C \to {}^{\omega}\mathbb{R}$ above a term to be omitted for an alternating differential form $\upsilon := \mathop{+}_{m=1}^{n} f_m \downarrow x_1 \wedge \ldots \wedge \overline{\downarrow x_m} \wedge \ldots \wedge \downarrow x_n$ of degree $\acute{n}$ on a cuboid $C = [a_1, b_1] \times \ldots \times [a_n, b_n] \subseteq {}^{\omega}\mathbb{R}^n$ where $\partial C := \mathop{\mp}_{m=1}^{n}\left(F_{a,m} - F_{b,m}\right)$ has the faces $F_{a,m} = [a_1, b_1] \times \ldots \times \{a_m\} \times \ldots \times [a_n, b_n]$ and $F_{b,m} = [a_1, b_1] \times \ldots \times \{b_m\} \times \ldots \times [a_n, b_n]$, then $\uparrow_C \downarrow \upsilon = \uparrow_{\partial C} \upsilon$.

Proof: The second fundamental theorem and Fubini's theorem (see above) give

$$\uparrow_C \downarrow \upsilon = \mathop{\mp}_{n=1}^{m} \uparrow_{a_n}^{b_n} \ldots \overline{\uparrow_{a_m}^{b_m}} \ldots \uparrow_{a_1}^{b_1}\left(f_m(x_1, \ldots, a_m, \ldots, x_n) - f_m(x_1, \ldots, b_m, \ldots, x_n)\right) \downarrow x_1 \wedge \ldots \wedge \overline{\downarrow x_m} \wedge \ldots \wedge \downarrow x_n$$

and

$$\frac{\downarrow f_m}{\downarrow x_m} \downarrow x_m \wedge \downarrow x_1 \wedge \ldots \wedge \downarrow x_{\acute{m}} \wedge \downarrow x_{\grave{m}} \wedge \ldots \wedge \downarrow x_n = -\underline{1}^{\widehat{m}} \frac{\downarrow f_m}{\downarrow x_m} \downarrow x_1 \wedge \ldots \wedge \downarrow x_n. \square$$

Remark: Stokes' theorem holds also for n-dimensional manifolds consisting of cuboids.

Advice

What can I tell you from my decades of mathematical experience?

Real progress in mathematics is only possible with unusual ideas that result from intensive consideration. The fewest problems are solved with luck. We need a mathematical pool from which we can use the mathematical tools for our problems. Success only comes with a healthy dose of tenacity, with which we can pursue a problem for decades if necessary, before we can solve it.

If we think deeply enough, it may require making whole theories obsolete. Therefore, it is necessary to challenge the so-called self-evident and to develop better solutions if we are not satisfied with the state of research. We have to develop a high sensitivity for the feasible and we must not be content with what we have achieved: we have to criticise ourselves.

It is not just the big and complicated ideas that solve a problem in the best way possible, but require long proofs. It is often the small, simple steps that bring success, as elaborate transformations have unfavourable complexity. We should also train the powers of observation and detach ourselves from existing literature, avoiding having to reinvent the wheel or miss important impulses.

If we cannot solve a mathematical problem iteratively, we should use automated computer-aided (proof) methods or be satisfied with a good approximation, even if the beauty and conciseness of the exact and closed representation is lost and the effort for the implementation is considerable if the solution to the problem is important enough or the required resources are available.

Often, the specification of error terms or numerical solutions (e.g., using interval arithmetic) is sufficient because high precision is not required. While certain statements are of value only in their exact form, it is more important to evaluate the problems according to their practicality and use, then to prioritise and finally satisfactorily resolve them.

There will always be numbers that cannot be represented by short sums or products of few (algebraic or transcendental) numbers, even if we have to (additionally) distinguish some of the numbers and to use them under a symbol. There is a good reason that we do not have the ability of the gods to survey entire levels of infinity at once and to filter out of them concise results.

Topology

In the following section, Set Theory is presupposed with the Euclidean norm $||\cdot||$.

Definition: A family of sets $\mathbb{Y} \subseteq \mathcal{P}(X)$ is called *topology* on $X \subseteq R$ if every intersection and union of sets of $\mathbb{Y}$ belongs apart from $\emptyset$ and X to $\mathbb{Y}$. The pair $(X, \mathbb{Y})$ is called *topological space*. If $\mathbb{Y} = \mathcal{P}(X)$, the topology is called *discrete*. A set $B \subseteq \mathbb{Y}$ is called a *base* of $\mathbb{Y}$ if every set of $\mathbb{Y}$ can be written as union of any number of sets of B. Every irreflexive relation $N \subseteq A^2$ founds a *neighbourhood relation (NR)* in $A \subseteq X$ for the underlying set X.△

Definition: If $(a, b) \in N$, a is said to be *neighbour* of or *neighbouring* to b. Particularly, an element $x \in A \subseteq X$ is called neighbour of an element $y \in A$ where $x \neq y$ if for all $z \in X$ and a mapping $d\colon X^2 \to \mathbb{R}_{\geq 0}$ holds: (1) $d(x, y) \leq \max\{\min\{d(x, z), d(z, x)\}, \min\{d(y, z), d(z, y)\}\}$ and (2) $d(z, z) = 0$. Here d is called *neighbourhood metric*. Let $P = R \cup V$ be the set of all *points* partitioned into *actual* points R and *virtual* points V for R, $V \neq \emptyset = R \cap V$. When R or V is clear from context, it can be omitted.△

Definition: The set $A' := R \setminus A$, where $A \subseteq R$, is called complement of A in R. A' can be called the exterior of A. All points of V (A) that have a neighbour in R $(A' \cup V)$ form the (inner) boundary ∂V (∂A) of V (A). Here $'$ takes precedence over ∂. When ∂ is applied successively beyond that, the argument is assumed to be without complement. The set $A^\circ := A \setminus \partial A$ is called the interior of A. If $\partial A \subseteq A$ is increased by the condition $\min\{d(x, y) : x \in A^\circ, y \in A'\} = \tilde{\mathrm{v}}$, let $A\ll := A \setminus \partial A$.△

Definition: A set $S \subseteq R$ (V) is said to be connected if there is for every partition of S into $Y \cup Z$ such that $Y, Z \neq \emptyset = Y \cap Z$: $\partial Y' \cap \partial Z \neq \emptyset \neq \partial Z' \cap \partial Y$. $S \subseteq R$ is moreover said to be simply connected if holds: Both $\partial Y' \cap \partial Z \cup \partial Z' \cap \partial Y$ for every partition into connected Y and Z and $S' \cup (\partial)V$ for S' as complement of S in R are connected for a connected $(\partial)V$. Let P and R be simply connected. Every $U \subseteq R$ is called neighbourhood of $x \in R$ if $x \in U^\circ$.△

Definition: If $\emptyset \neq \mathbb{D} \subseteq (X, \mathbb{Y})$ holds, a connected $\mathbb{D}$ is called *domain*. The set of h-$S \subseteq \mathbb{R}^m$ for $m \in \mathbb{N}^*$ is *n-dimensional*, where $m \geq n \in \mathbb{N}^*$, if and only if it contains at least one n-cube with edge length $h \in \mathbb{R}_{>0}$ and maximum n. Let ${}^1\dot{\mathrm{R}}^n$ the *unit ball* with the special case *unit disk* ${}^1\dot{\mathrm{R}}^2$. *Midpoints* a of n-balls and n-cubes may be denoted in brackets as (a) after those. A set has dimension ${}_\iota\mathrm{n}$ if its elements consist of n maximal cubes of edge length ι.△

Examples: The base for $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{A}_\mathbb{R}$, $\mathbb{A}_\mathbb{C}$, $\mathbb{R}$ and $\mathbb{C}$ is precisely each related discrete topology. The boundary of every n-ball with $n \geq 2$ is only connected, itself it is simply connected. For $n = 1$ both are not connected. For $n \geq 2$ und $r \in \mathbb{R}_{>0}$, the n-*Torus* ${}^\mathrm{r}\mathrm{T}^\mathrm{n} := \left(\partial^\mathrm{r}\dot{\mathrm{R}}\right)^\mathrm{n}$ is only connected.

Theorem: For $n \in \mathbb{N}_{\geq 2}$, no finite decomposition of an n-ball can be reassembled giving an n-cube, since finitely many convex boundaries cannot have the same order of concave counterparts.□

Theorem: Traversing up to nine cells of a tic-tac-toe grid renders the singleton the only connected set with fixed point property (invalid hairy ball theorem).□

Definition: A function between two topological spaces is said to be *continuous* if for every point that is mapped holds: for every neighbourhood of the image of this point there is a neighbourhood of the point whose image lies completely in the neighbourhood of the image of this point.△

Remark: The suggestive terms compactness and countability (possibly misleading) are not used since they are not given for infinite sets like $\mathbb{R}$ and $\mathbb{N}$ resp. For all contracting deformations, points are to be removed from the target set as the contraction specifies. Only then the (generalised) Poincaré conjecture holds.

Remark: The neighbouring boundary points of the conventional closed [0, 1] and the conventional open]0, 1[especially have not the Hausdorff property (see [782], p. 83). So not every metric space can be a Hausdorff space or normal, and (pre-) regular spaces are limited. The spaces $\mathbb{C}^n$ and $\mathbb{R}^n$ with $n \in {}^\omega\mathbb{N}^*$ have therefore only the Fréchet topology (cf. [915], p. 62). The situation is, however, different in partially imprecise conventional mathematics.

Scientific Computing

The following section presupposes Set Theory and Nonstandard Analysis. Let $\varepsilon \in [\tilde{\nu}, 1 - \tilde{\nu}]$.

Definition: Let $f_n^*(z) = (\eta_n z)$ *sisters* of the TS $f(z)$ centred on 0 on the domain $\mathbb{D} \subseteq {}^{\omega}\mathbb{C}$ where $m, n \in {}^{\omega}\mathbb{N}^*$ and $\eta_n^m := \underline{1}^{2^{\lceil m/n \rceil}}$. Then let $\delta_n^* f = \check{f}\text{-}\check{f}_n^*$ the *halved sister distances* of f. Let $\mu_n^m := m!n!/(m+n)!$ and $\widetilde{(-n)}! := 0$. For $j, k \in {}^{\omega}\mathbb{N}$, $f \in C_{\pi}^{j+2}$ and *Fourier coefficients* $c_k := \widetilde{\hat{\pi}} \uparrow_{-\pi}^{\pi} f(t)\tilde{\epsilon}^{\underline{kt}} \downarrow t$ (cf. [739], p. 358 ff.) the series $\mathop{+}_{k=-\omega}^{\omega} c_k \epsilon^{\underline{kt}}$ is called *Fourier series*. On the level of TS, μ and η form the μ-η-*calculus* △

Remark: The latter allows representing integrals and derivatives easily and finitely closed (cf. [473], p. 165 f.). A conversion to other period lengths than $\hat{\pi}$ is easily possible (see [739], p. 364).

Example: Re $c \in [\tilde{\nu}, 1 + \tilde{\nu}[$, $c \in {}^{\omega}\mathbb{C}$ and $\varepsilon := \tilde{2}^j$, $j \in {}^{\omega}\mathbb{N}^*$ imply $\zeta(n + c) = 2^{jn}\,\tilde{n} \mathop{+}_{k=1}^{n} \delta_n^* v_c(\tilde{2}^j u^k)$ for $z \in {}^{1-\tilde{\nu}}\dot{C} \subset \mathbb{D}$ (cf. [474], p. 42) and $v_c(z) := \mathop{+}_{m=1}^{\omega} \zeta(m + c) z^m = z \mathop{+}_{m=1}^{\omega} \tilde{m}^c z^{\widetilde{-m}}$. □

Example: A Richardson extrapolation determines the digamma function ψ and

$$\zeta(\dot{n}) = \varepsilon^{\widetilde{n}}\hat{n} \mathop{+}_{k=1}^{n} \left(\psi(\varepsilon u^k \underline{1}^{2\tilde{n}}) - \psi(\varepsilon u^k)\right) + O(\varepsilon^{\hat{n}})$$

for $n = 2$ and $\varepsilon = \tilde{2}^{13}$ as $\zeta(3) = 2^{24} \mathop{+}_{k=1}^{2} \left(\psi(\underline{\varepsilon} u^k) - \psi(\varepsilon u^k)\right) + O(10^{-16})$.

Representation theorem for integrals: The TS (see below) $f(z) \in \mathcal{O}(\mathbb{D})$ centred on 0 for $\mathbb{D} \subseteq {}^{\omega}\mathbb{C}$ gives for $\dot{m}$, $n \in {}^{\omega}\mathbb{N}^*$

$$\uparrow_0^z \ldots \uparrow_0^{\zeta_2} f(\zeta_1) \downarrow \zeta_1 \ldots \downarrow \zeta_n = \widetilde{n!}\, f(z\mu_n) z^n. \square$$

Example: For the TS $f(x), g(x) \in {}^{\omega}\mathbb{R}$, it holds that $\uparrow_0^x f(v) \downarrow v \uparrow_0^x \uparrow_0^y g(v) \downarrow v \downarrow y = f(x\mu_1)\check{g}(x\mu_2)x^3$.

Representation theorem for derivatives: For ${}^{\tilde{\nu}}\dot{C} \subset \mathbb{D} \subseteq {}^{\omega}\mathbb{C}$, n-th unit roots, TS

$$f(z) := f(0) + \mathop{+}_{m=1}^{\omega} \widetilde{m!}\, {}^m f(0) z^m,$$

$\varepsilon := \tilde{2}^j \tilde{r}$, $j \in {}^{\omega}\mathbb{Z}$, $n = \epsilon^{\sigma} \in {}^{\omega}\mathbb{N}^*$, $u := \epsilon^{\tilde{n}\underline{\hat{n}}}$ and f's radius of convergence $r \in {}^{\nu}\mathbb{R}_{>0}$ imply

$${}^n f(0) = 2^{jn}\acute{n}! \mathop{+}_{k=1}^{n} \delta_n^* f(\tilde{2}^j u^k). \square$$

Universal multistep theorem: For $n \in {}^{\nu}\mathbb{N}_{\leq p}$, $k, m, p, q \in {}^{\nu}\mathbb{N}^*$, $\downarrow x \in]0, 1[$, $x \in [a, b] \subseteq {}^{\omega}\mathbb{R}$, $y : [a, b] \to {}^{\omega}\mathbb{R}^q$, $f : [a, b] \times {}^{\omega}\mathbb{R}^{q \times n} \to {}^{\omega}\mathbb{R}^q$, $g_k(\vec{x}) := g_{\acute{k}}(x)$, and $g_0(a) = f(\bar{a}, y_0, \ldots, y_{\acute{n}})$, the TS of the initial value problem ${}^1y(x) = f(x, y((\rightharpoonup)^0 x), \ldots, (y(\rightharpoonup)^{\acute{n}} x))$ of order n implies

$$y(\vec{x}) = y(x) + \downarrow\vec{x} \mathop{\pm}_{k=1}^{p} \left(g_{p-k}(\vec{x}) \mathop{+}_{m=k}^{p} \widetilde{m!} \binom{\acute{m}}{\acute{k}}\right) + O((\downarrow\vec{x})^{\grave{p}}). \square$$

Theorem for (anti-) derivatives of TS: For $j \in {}^{\omega}\mathbb{Z}$, $q = \tilde{\varepsilon}(z - a)$, $a \in \mathbb{D}$ and $k, m \in \mathbb{N}_{<n}$, modular arithmetic (cf. [973], p. 302 – 311) and n-th unit roots result in the corresponding *DFT form*:

$${}^m f_n(z) := \tilde{n}(q^j)^T \left(\delta_{jk} \widetilde{\varepsilon q}^m j!/(j-m)!\right)\left(\tilde{u}^{jk}\right)\left(f(\varepsilon u^k + a)\right) + O(\varepsilon^n). \square$$

Reversion theorem for TS: For $y \in f(\mathbb{D})$, $y(a) = b$ and ${}^1y(a) \neq 0$, Bürmann's theorem yields ([1076], p. 129 ff.):

$$f^{-1}(y) = a + \tilde{n} \mathop{+}_{m=1}^{n} \tilde{m}\tilde{\varepsilon}^{\acute{m}}(y-b)^m \left(\tilde{u}^{\acute{m}k}\right)\left(f\left(\varepsilon u^k + a\right)^{-m}\right) + O(\varepsilon^n). \square$$

Remark: Put $y = 0$ for a sufficiently small $|a - z_0|$ such that $z_0 \in {}^{\omega}\mathbb{C}$ is a zero of $f(z) \in {}^{\omega}\mathbb{C}$.

Theorem for derivatives of Fourier series: For $f \in C_{\pi}^{m+2}$ (cf. [739], p. 358 ff.), $m \in {}^{\omega}\mathbb{N}$, $j \in [-\acute{n}, n] \cap \mathbb{Z}$, $t \in [-\pi, \pi]$ and $k \in N_{<\hat{n}}$, the following DFT form is implied allowing integration of individual terms:

$$^{m}f_n(t) := \tilde{n}\left(u^{\tilde{\pi}jnt}\right)^T \left(\delta_{(j+\acute{n})k}\underline{j}^m\right)\left(\tilde{u}^{jk}\right)\left(\check{f}(\tilde{n}\pi k\text{-}\pi)\right) + O(\tilde{n}). \square$$

Conclusion: Supporting points $kr := \tilde{n}\pi k$ of the smooth $f(kr)$ yield for j like k interpolating in $\mathcal{O}(\sigma n)$:

$$^{m}f_n(t) := \tilde{n}(u^{\check{r}jt})^T \left(\delta_{jk}\underline{j}^m\right)\left(\tilde{u}^{jk}\right)\left(\check{f}(kr)\right). \square$$

Theorem (DFT fixed-point method): Every $z \in {}^{\omega}\mathbb{C}$ of an arbitrary m-polynomial $p(z) = 0$ with $m \in [2, n] \cap \mathbb{N}$ for $n := 2^r$, $r \in {}^{v}\mathbb{N}$ and coefficients from ${}^{v}\mathbb{C}$ can be (initiating) determined also in $\mathcal{O}(\sigma n)$.

Proof and algorithm: Let $U = (\tilde{u}^{jk})$ for $j, k \in \mathbb{N}_{<n}$, $u := \epsilon^{\tilde{n}\tilde{\pi}}$, $q := 2z$ and $s_k := p(\tilde{2}u^k)$. A simple transform achieves $|q| < \tilde{2}$ for all zeros ζ of $p(z)$ and $p(0) = 1$. It follows from $p(z) = \tilde{n}(q^j)^T Us = \tilde{n}\mu^T s = 0$ the simplified iteration $\mu^* = U_1^{-T}\mu\, U((\delta_{jk}\tilde{u}^j)\, U^{-1}\mu - (U_{\acute{n}}^{-1}\mu + \beta s^T\mu, \beta s^T\mu, \ldots, \beta s^T\mu)^T)$ with Kronecker delta δ_{jk}, starting point $q := \tilde{2}$ and $\beta \in {}^{v}\mathbb{C}$ such that each time $\|\mu^*\text{-}\mu\|$ is roughly halved and $\mu^T s = 0$ holds. Finish by polynomial division where $m > 2$.$\square$

Remark: The identity instead of δ_n^* still provides arbitrarily precise approximations for the ${}^{n}f$. The last theorems are equally valid for multidimensional TS and Laurent series. The error is $\mathcal{O}(\varepsilon^n)$ instead of $\mathcal{O}(\varepsilon)$ for analogously defined m-dimensional DFT forms of the TS with $\binom{m+n}{n}$ derivatives where the effort is comparable. A good choice for the *crown points* $\varepsilon u^m + a$ around the *crown centre* a is $\varepsilon = \tilde{2}$ and $n = 64$.

Approximation theorem: The ${}^{s}f(x) \in {}^{\omega}\mathbb{R}$ for $x \in {}^{\omega}\mathbb{R}$ and $m, n \in {}^{v}\mathbb{N}$ as above allow computing the interpolant

$$g(x) := +_{r=0}^{\acute{m}}\chi_{]x_{\grave{r}},x_{\grave{r}}[}(x)\ \left((x_{\grave{r}}\text{-}x)p_r(x) + (x\text{-}x_r)p_{\grave{r}}(x)\right)/(x_{\grave{r}}\text{-}x_r) + +_{r=0}^{m}\chi_{\{x_r\}}(x)p_r(x)$$

for $p_r(x) := +_{s=0}^{n}{}^{s}f(x_r)(x\text{-}x_r)^s/s!$ in $\mathcal{O}(\sigma mn)$ where ${}^{s}f(x_r) = {}^{s}g(x_r)$ holds for every $x_r \in {}^{\omega}\mathbb{R}$. Replace in the complex case ${}^{\omega}\mathbb{R}$ by ${}^{\omega}\mathbb{C}$ and put $x = \gamma(t) \in {}^{\omega}\mathbb{C}$ for the path $\gamma(t)$ where $t \in {}^{\omega}\mathbb{R}$.$\square$

Theorem (differential equations): Let $d(z) \in {}^{\omega}\mathbb{C}^{n+2}$ vector of DFT forms for $n \in {}^{\omega}\mathbb{N}$. If the system ${}^{\grave{n}}y = \left({}^{n}y, {}^{\acute{n}}y, \ldots, y, 1\right)d(z) \in {}^{\omega}\mathbb{C}$ where (not) $\left({}^{n}y(a), {}^{\acute{n}}y(a), \ldots, y(a)\right)^T \in {}^{\omega}C^{\grave{n}}$ and $a, z \in {}^{\omega}\mathbb{C}$ has the solution y_0, this can be determined in $\mathcal{O}(n^3)$ giving the error mentioned above.$\square$

Remark: If function values of crown points are evaluated and terminating TS or polynomials are used, both a polynomial division may be executed and non-linear or partial (systems of) differential equations may be solved via DFT forms.

Example: For $b = (f(\varepsilon u^0 + a), \ldots, f(\varepsilon u^{\acute{n}} + a), 1)^T \in {}^{\omega}C^{\grave{n}}$ and an upper triangle matrix $T(z) \in {}^{\omega}C^{\grave{n}\times\grave{n}}$, ordinary differential equations have only solutions if $T(z)b = 0$ holds (Cholesky decomposition!).

Definition: To increase precision, let according to the *trapezoidal rule* ${}^{T}\uparrow_{z\in A} f(z)\downarrow z := +_{z\in A}\tilde{2}\left(f(z) + f(\vec{z})\right)(\vec{z}\text{-}z)$ and according to the *midpoint rule* ${}^{M}\uparrow_{z\in A} f(z)\downarrow z := +_{z\in A} f\left(\tilde{2}(z + \vec{z})\right)(\vec{z}\text{-}z)$. $\triangle$

Series theorem for integrals: For $c \in [0, a]$, $f \in C^n$ and $n, \hat{p}, r\ (= 2^p), t\ (= 2^v), \hat{v} \in {}^{v}2\mathbb{N}^*$, DFT forms of TS show

$$\uparrow_0^a f(w)\downarrow w = \uparrow_0^{\check{t}a} +_{s=1}^{t} f(x + \acute{s}\check{t}a)\downarrow x = \uparrow_0^1 g(y)\downarrow y = +_{\check{q}=1}^{\check{r}} +_{\check{m}=0}^{\check{n}\text{-}1}\widetilde{\grave{m}!}\ {}^{m}\hat{g}(\check{r}\acute{q})\check{r}^{\grave{m}} + O\left(\widetilde{\grave{n}!}\ {}^{n}g(\check{a}c)\check{r}^n\right). \square$$

Remarks: DFT equivalents may replace any ${}^{m}g$ for $g(y) := af(ay)$ and $a > 0$. Transforming $z := \epsilon^{\pm x}$ makes here infinite bounds of integration finite. A finite integral decreases the remainder's modulus sufficiently. The midpoint rule is here more advantageous than the trapezoidal rule.

Example: For ${}_{\mp}g(\vartheta) := \check{\pi}(1\text{-}\varepsilon^2\sin^2(\check{\pi}\vartheta))^{\mp\tilde{2}}$, complete elliptic integrals of the first and second kind are

$$\uparrow_0^1 {}_{\mp}g(\vartheta)\downarrow\vartheta = {}_{\mp}g(\check{1}) + \widetilde{24}\,{}_{\mp}^{2}g(\check{1}) + \widetilde{1920}\,{}_{\mp}^{4}g(\check{1}) + O\left(\widetilde{9!}\,{}_{\mp}^{6}g(\check{1})\right).$$

Second Euler-Maclaurin formula: For $f(\check{q}) = g(\check{r}\acute{q})$ and $k = \grave{a} = \grave{r}$, the preceding theorem yields in $\mathcal{O}(\sigma n)$

$$+_{\check{q}=1}^{\check{r}} f(\check{q}) = \uparrow_{\check{1}}^{\check{k}} f(x) \downarrow x + +_{\check{m}=1}^{\check{n}-1} H_m \left({}^{\acute{m}}f(\check{k}) - {}^{\acute{m}}f(\check{1}) \right) + O\left(H_n \left({}^{\acute{n}}f(\check{k}) - {}^{\acute{n}}f(\check{1}) \right) \right).$$

Proof: For $h(x) = x/\sin x$ and $H_m := i^m \widetilde{m!}\ {}^m h(0) = \widetilde{m!}\, B_m(2-2^m) \to 2\underline{\tilde{\pi}}^{-m}$,

$$+_{\check{q}=1}^{\check{r}} g(\widetilde{r\acute{q}}) = \check{r} \uparrow_0^1 g(y) \downarrow y - +_{\check{q}=1}^{\check{r}} +_{\check{m}=1}^{\check{n}-1} \widetilde{m!}\ {}^m g(\widetilde{r\acute{q}}) \tilde{r}^m + O(\tilde{n!}\ {}^n g(\tilde{a}c) \tilde{r}^{\acute{n}})$$

results in the claim by inserting and combining the factorials that are counted depending on partitions where B_m are Bernoulli numbers and $B_{\grave{m}} = 0$, $B_0 = 1$ as well as $B_1 = -\tilde{2}$.□

First Euler-Maclaurin formula: Mathematical induction for n shows also that (cf. [1096], p. 193 f.)

$$+_{\check{q}=0}^{\check{r}} f(\check{q}) = \uparrow_0^{\check{r}} f(x) \downarrow x + \check{f}(\check{r}) + \check{f}(0) + +_{\check{m}=1}^{\check{n}-1} \widetilde{m!}\, B_m \left({}^{\acute{m}}f(\check{r}) - {}^{\acute{m}}f(0) \right) + O\left(\tilde{n!}\, B_n \left({}^{\acute{n}}f(\check{r}) - {}^{\acute{n}}f(0) \right) \right). \square$$

Number Theory

This section requires Set Theory, Topology and Nonstandard Analysis. Let $k \in \mathbb{N}$.

Prime number theorem: For $\pi(x) := |\{p \in \mathbb{P}_{\leq x} : x \in {}^{\omega}\mathbb{R}\}|$, it holds that $\pi(\omega) = {}_{\epsilon}\widetilde{\omega}\ \omega + \mathcal{O}\left({}_{\epsilon}\omega\ \omega^{\tilde{2}}\right)$.

Proof: From intervals of fix length $y \in {}^{\omega}\mathbb{R}_{>0}$, $\check{y}$ set-2-tuples of prime numbers are formed such that the first interval has the unchanged representative prime density and the second interval is empty, then the interval with the second most prime density is followed by the second least one etc. The Stirling formula (see Nonstandard Analysis) suggests the prime gap $n = \epsilon^{\sigma} = \mathcal{O}({}_{\epsilon}n!)$ for $n \in {}^{\omega}\mathbb{N}_{\geq 2}$.

For induction basis $n = 2$ resp. 3, the hypothesis states the first interval to contain $x_n/{}_{\epsilon}x_n$ primes and $x_4 \in [2, 4[$. Stepping from x_n to x_n^2 finds $\pi(x_n^2) = \pi(x_n)\check{x}_n$ primes only from $\pi(x_n) = x_n/{}_{\epsilon}x_n$. The average prime gap is ${}_{\epsilon}x_n$, the maximal one ${}_{\epsilon}x_n^2$ (see below) and the maximal x_n^2 to x_n behaves like ω to $\omega^{\tilde{2}}$.□

Remark: Replacing 2 by $m \in {}^{\omega}\mathbb{N}_{>2}$ for $\widetilde{m}y^{\acute{m}}$ set-m-tuples gives the same result. Induction and the sieve of Eratosthenes show by the Dirichlet prime number theorem both infinitely many prime and composite Mersenne numbers $M_n := 2^n - 1$ for $n \in {}^{\omega}\mathbb{N}^*$ (see [455], p. 174 f. and 354 – 365).

Singmaster's theorem: There are maximally 8 distinct binomial coefficients of the same value > 1.

Proof: The existence is clear due to $\binom{3003}{1} = \binom{78}{2} = \binom{15}{5} = \binom{14}{6}$ and the structure of Pascal's triangle. With $p \in {}^{\omega}\mathbb{P}$, $a, b, c, d \in {}^{\omega}\mathbb{N}^*$, $\hat{a} \leq r := p - b$, $\hat{a} < \hat{c} \leq n := p - d$, $b < d$ and $s \notin \mathbb{P}$ for every $s \in [\max(r - \acute{a}, \grave{n}), r]$, Stirling's formula $n!^2 \sim \pi(\hat{n} + \tilde{3})(\tilde{\epsilon}n)^{\hat{n}}$ and the prime number theorem imply $\omega\binom{r}{a} \leq {}_{\epsilon}\omega\binom{n}{c}$ for $p \to \omega$.□

Giuga's theorem: Every number $n \in {}^{\omega}\mathbb{N}_{\geq 2}$ is prime if and only if $\mathop{+}_{k=1}^{\acute{n}} k^{\acute{n}} \equiv -1 \bmod n$.

Proof: Fermat's (little) theorem settles the case $n \in {}^{\omega}\mathbb{P} \cup {}^{\omega}2\mathbb{N}^*$. Otherwise, the harmonic and geometric mean H_n resp. G_n imply $\mathop{+}_{p \in {}^{\omega}\mathbb{P}} \tilde{p} - \mathop{\times}_{p \in {}^{\omega}\mathbb{P}} \tilde{p} = m/H_n - G_n^{-m} = c \in {}^{\omega}\mathbb{N}^*$ (cf. D. Borwein, J. M. Borwein, P. B. Borwein and R. Girgensohn: *Giuga's Conjecture on Primality*. Amer. Math. Monthly 103:40-50, 1996, p. 3 f.). This contradicts $c < 1$ due to $H_n(m) \neq H_n = H_n(m, n) = m/(c + \tilde{n}) < n^{\tilde{m}} = G_n$.□

Prime gap theorem: For the set M_g of mediate prime gaps, $M_g \supset {}^{\omega}2\mathbb{N}^* \cup \{1\}$ holds.

Proof by induction: The claim states that beside 1 the mediate prime gaps m_p exist from 2 to $\acute{p}$. It is true for primes $p \in \{2, 3\}$. By stepping from $p \longrightarrow p + 2$, the prime number theorem permits no greater prime gap than those occurred as m_{p+2}. Hence $\grave{p}$ exists, too.□

Goldbach's theorem: Every even whole number > 2 is the sum of two primes.

Proof: For $\widehat{m} + \hat{n} = p_{m+r,n-r} + q_{m+r,n-r} + r$, $r \in \{0, 2, \dots, \max(g(n))\}$, it follows alike $\widehat{m} + \hat{n} = p_{m+s,n-s} + q_{m+s,n-s} + s$, $s \in \{0, 2, \dots, \max(g(n)) + 2\}$. This implies $\widehat{m} + \hat{n} + 2 = p_{\grave{m}+r,\grave{n}-r} + q_{\grave{m}+r,\grave{n}-r} + r$, $r \in \{0, 2, \dots, \max(g(\grave{n}))\}$. Induction yields then the claim by the previous theorem.□

Conclusion by Fortune: The previous theorem yields Bertrand's postulate from ([455], p. 34 f.) for all $n \in {}^{\omega}\mathbb{N}^*$ and $\min\{j \in {}^{\omega}\mathbb{N}_{\geq 2} : m = p_n\# + j \in {}^{\omega}\mathbb{P}\} = p_{\grave{n}+k}$ due to $j \perp m$ where p_n is the n-th prime.□

First conclusion by Hardy-Littlewood: The number of twin primes is given by $\pi_2(n) \sim C_2\ \hat{x}\ {}_{\epsilon}\widetilde{x^2}$.□

Second conclusion by Hardy-Littlewood: The number of prime n-tuples for every $n \in {}^{\omega}\mathbb{N}_{\geq 2}$ is infinite.□

Conclusion by Cramér: Every $n \in {}^{\omega}\mathbb{N}_{\geq 2}$ and $s = \mathop{+}_{m=0}^{\omega} {}_{\epsilon}m(n)$ have $p \in {}^{\omega}\mathbb{P}$ such that $\lfloor \sigma n \rfloor \leq p \leq \lceil {}_{\epsilon}s\ s \rceil$.□

Theorem of second Hardy-Littlewood conjecture: For $m, n \in {}^{\omega}\mathbb{N}_{\geq 2}$, $\pi(m + n) \leq \pi(m) + \pi(n)$ holds.

Proof by induction for n: Cases $\pi(m + \grave{n}) = \pi(m + n)$ resp. $\pi(\grave{n}) = \pi(n) + 1$ and $\pi(m + \grave{n}) = \pi(m + n) + 1$ are clear. For $\pi(\grave{n}) = \pi(n)$, the last case implies the claim for $m := n + k$ and $\pi(n) = {}_{\epsilon}\widetilde{n}\ n + \mathcal{O}\left(\sigma n^{\tilde{2}}\right)$ by $\pi(\hat{n} + k) + 1 \leq \pi(n + k) + \pi(n)$ and $\pi(4) \leq 2\pi(2)$ and so on due to

$$(n+k)({}_\epsilon(\hat{n}+k)-{}_\epsilon(n+k))\sigma+n({}_\epsilon(\hat{n}+k)-\sigma){}_\epsilon(n+k)\geq {}_\epsilon(\hat{n}+k){}_\epsilon(n+k)\sigma.\square$$

Coefficient theorem for ω-ANs: No zero of normalised irreducible polynomials and series with at least one $a_k \notin {}^\omega\mathbb{Z}$ is ω-AN, since these are pairwise distinct and uniquely determined.□

Bounding theorem for ω-ANs: No $z \in \mathbb{C}^*$ such that $|z| \notin [\tilde{\omega}, \omega]$ is ω-AN.

Proof: In a polynomial or series equation, put $a_m = 1$ and $a_k = -\acute{\omega}$ for $k < m$, then the real case follows from the GS formula after taking the reciprocal. The exact limit value can be found by replacing ω by $\omega(m) = \omega - \acute{\omega}/\omega(m)^m$. The complex case is solved by putting i. a. $x = \grave{y}\omega$ for $y \in {}^\omega\underline{\mathbb{R}}^*$.□

Conclusions: For every $z \in \mathbb{R} + \underline{\mathbb{R}}$ where $|z| \notin \mathbb{B}$ and $\eta := z^{\grave{\omega}}$, the GS is $\bigplus_{n=0}^{\omega} z^n = \acute{\eta}/\acute{z} \notin {}^\omega_\omega\mathbb{A}_\mathbb{C}$. Putting $k = \omega^2!$ implies $\Gamma(z) := k!\, k^z/(z\grave{z} \dots (z+k)) \notin {}^\omega_\omega\mathbb{A}_\mathbb{R}$ for all $z \in {}^\omega\mathbb{R} \setminus -{}^\omega\mathbb{N}$. Euler's number $\epsilon = (1+\tilde{\omega})^\omega$ implies $\epsilon = (k\omega + 1)/\omega!$ for $k > \omega$ (exponential series).□

Counting theorem for ANs: For the Riemann zeta function ζ and the average number $z(m)$ of zeros of a $\grave{m}$-polynomial or $\grave{m}$-series, the ANs asymptotically have for $\check{\kappa} = n$ the number

$$\mathbb{A}(m,n) = \widetilde{\zeta(\grave{m})} z(m) \grave{\kappa}^m \big(n + \mathcal{O}(\sigma)\big).$$

Proof: The case $m = 1$ has by ([455], p. 323) the error term $\mathcal{O}(\sigma n)$ and represents the number $4 \bigplus_{k=1}^{n} \varphi(k) - 1$ by the φ-function. For $m > 1$, the divisibility conditions neither change the error term $\mathcal{O}(\sigma)$ nor the leading term. By $1/\zeta(\grave{m}) = \bigtimes_{i=1}^{n} (1 - \tilde{p}_i^{\grave{m}})$ (GS!), which absorbs multiples of prime p_i, polynomials or series such that $\gcd(a_0, a_1, \dots, a_m) > 1$ are excluded.□

Examples: For $m = 1$, there are $3(n/\check{\pi})^2 + \mathcal{O}(\sigma n)$ real solutions. For $m = 2$, $\check{9}n^3/\zeta(3) + \mathcal{O}(\sigma n^2)$ real solutions arise, since a real polynomial of degree 2 has two real zeros with probability $\frac{9}{16}$ by the quadratic formula. For $a_m = 1$, there are $z(m)\grave{\kappa}^{\acute{m}}(\kappa + \mathcal{O}(\sigma))$ algebraic integer solutions.

Fundamental theorem of algebra: Every non-constant polynomial $p \in {}^{(\omega)}\mathbb{C}$ has at least one complex root.

Indirect proof: By performing an affine substitution of variables, reduce to the case $\widetilde{p(0)} \neq \mathcal{O}(\iota)$. Suppose that $p(z) \neq 0$ for all $z \in {}^{(\omega)}\mathbb{C}$. Since $f(z) := \widetilde{p(z)}$ is holomorphic, it holds that $f(\tilde{\iota}) = \mathcal{O}(\iota)$. By the mean value inequality $|f(0)| \leq |f|_\gamma$ (see [473], p. 160) for $\gamma = \partial^r\dot{\mathbb{C}}$ and arbitrary $r \in {}^{(\omega)}\mathbb{R}_{>0}$, and hence $f(0) = \mathcal{O}(\iota)$, which is a contradiction (and hence exactly $z(m) = m$ holds).□

Remark: In the real case, $z(m)$ is asymptotically equal to ${}_\epsilon m/\check{\pi} + \mathcal{O}(1)$ (Kac, Mark: *On the Average Number of Real Roots of a Random Algebraic Equation*. II.; Bull. Amer. Math. Soc. 49 (4); 1943; 314 – 320).

Conclusion: For $m = n = \acute{\nu} =: \epsilon^\sigma$, it is true that $|{}^\nu\mathbb{A}_\mathbb{R}| = \tilde{\pi}\sigma\grave{\kappa}^{\acute{\nu}}(\hat{\nu} + \mathcal{O}(\sigma))$ and $|{}^\nu\mathbb{A}_\mathbb{C}| = \grave{\kappa}^\nu(\check{\nu} + \mathcal{O}(\sigma))$. □

Bunyakovsky theorem: If $a_n > 0$ holds for a minimal polynomial $p \in {}^{(\omega)}\mathbb{C}$ of degree n where $p(i) \perp p(j)$ for every ${}^\omega\mathbb{N}^* \ni i < j \in {}^\omega\mathbb{N}^*$, then it has infinitely many primes as values.

Proof: The Dirichlet prime number theorem yields for $m \in {}^\omega\mathbb{N}^*$ infinitely many primes $q_m = bm + c$ where $b \perp c \in {}^\omega\mathbb{N}^*$ are fix. Vice versa there are infinitely many m such that $d_m := p(m) = \tilde{m}(q_m - c)$. By rearranging the latter formula, the claim follows by mathematical induction for the polynomial degree n.□

Theorem: The BBP series $s_k := \bigplus_{n=1}^{\omega} p(n)\widetilde{q(n)}b^n$ implies, where $b \in {}^\omega\mathbb{N}_{\geq 2}$ and integer polynomials resp. series p and q with $q(n) \neq 0$ and $\deg(p) < \deg(q)$, that $s_k \notin {}^\omega_\omega\mathbb{A}_\mathbb{R}$ due to $\mathrm{den}(s_k) \geq b^m > \omega$ with $m \in \mathbb{N}^*$.□

Theorem: The maximum distance between two neighbouring real ω-ANs is $\Omega/\acute{\omega}$ for the not ω-algebraic omega constant $\Omega = \tilde{\epsilon}^\Omega = W(1)$ (see below Lambert-W function).

Proof: The distance between two real ω-ANs is largest around the points ± 1. A real ω-algebraic x approximates 1 satisfying the polynomial or series equation $\acute{x}x^{\acute{m}}\acute{\omega} = 1$ for $x > 1$ or $x^m = -\acute{x}\acute{\omega}$ for $x < 1$.□

Approximation theorem for real ω-ANs: The average asymptotic error to approximate every real ω-AN of degree $n > 1$ by a real ω-AN of degree $m < n$ is $|{}^{\omega}\mathbb{Z}|^{-m}{}_{\epsilon}\widetilde{m}\zeta(\dot{m})\breve{\pi}$.

Proof: The number of ω-ANs approximately evenly distributed between fixed limits increases in ${}^{\omega}\mathbb{R}$ by a factor of approximately $|{}^{\omega}\mathbb{Z}|$ per degree. The error corresponds to the distance between ω-ANs. The non-real ω-ANs are less dense.□

Conclusion: Two distinct real ω-ANs have an average distance of at least $|{}^{\omega}\mathbb{Z}|^{-\dot{\omega}}{}_{\epsilon}\widetilde{\omega}\pi$. Determining this minimum distance exactly requires an infinite non-linear non-convex optimisation problem to be solved. Therefore, the ν-ANs have an approximate order of $\mathcal{O}(\nu)$. This disproves Roth's theorem, which begins with false assumptions.□

Gelfond-Schneider theorem: It holds $a^b \notin {}^{\omega}_{\omega}\mathbb{A}_{\mathbb{C}}$ where $a, c \in {}^{\omega}\mathbb{A}_{\mathbb{C}} \setminus \mathbb{B}$ and infinitesimal $\varepsilon, b \in {}^{\omega}\mathbb{A}_{\mathbb{C}} \setminus {}^{\omega}_{\omega}\mathbb{R}$.

Indirect proof: The minimal polynomials p (and q) of c^r resp. $c^{r\pm\varepsilon} = a^b$ for maximal $r \in {}^{\omega}_{\omega}\mathbb{R}_{>0}$ and $f = p\ (q)$ lead to the contradiction ${}^1f(c^{r(\pm\varepsilon)}) \neq 0 = (f(c^r) - f(c^{r\pm\varepsilon}))\ /\ (c^r - c^{r\pm\varepsilon}) = {}^1f(c^{r(\pm\varepsilon)})$.□

The greatest-prime criterion (GPC) for ω-ANs: If $r := \widetilde{ap}b \pm \tilde{s}t$ is an irreducible fraction, where a, b, s and t are natural numbers, $abst \neq 0$, $a + s > 2$, and the (second-)greatest prime number $p \in {}^{\omega}\mathbb{P}$, $p \nmid b$ and $p \nmid s$, then $r \notin {}^{\omega}\mathbb{A}_{\mathbb{R}}$, since the prime number theorem implies $\mathrm{den}(\widetilde{aps}(bs \pm apt)) \geq \hat{p} \geq \widehat{\omega} - ({}_{\epsilon}\omega\ \omega^{\tilde{2}}) > \omega$.□

Theorem: The constants of Catalan (G), Gieseking ($\pi\ {}_{\epsilon}\beta$), Smarandache (S_1) and Taniguchi (C_T) are not ω-algebraic because of the GPC.□

Theorem: It holds $\pi \notin {}^{\omega}\mathbb{A}_{\mathbb{R}}$ provided that its different representations are accepted as Wallis product, or product using the gamma function with value $\tilde{2}$ (see Nonstandard Analysis), by alternatively applying the GPC to the Leibniz series, or the TS of $\arcsin(x)$ at $x = 1$.□

Theorem: The constants of Artin (C_{Artin}), Baxter (C^2), Chaitin (Ω_F), Champernowne (C_{10}), Copeland-Erdős (C_{CE}), Erdős-Borwein (E), Feller-Tornier (C_{FT}), Flajolet and Richmond (Q), Glaisher-Kinkelin (A), Heath-Brown-Moroz (C_{HBM}), Landau-Ramanujan (K), Liouville ($£_{Li}$), Murata (C_M), Pell (P_{Pell}), Prouhet-Thue-Morse (C_{PTM}), Sarnak (C_{sa}) and Stephen (C_S) as well as the Euler resp. Landau totient constant (ET resp. LT), the twin prime constant (C_2) and the carefree constants (K_1, K_2 and K_3) are not ω-ANs, since simplifying cannot remove a certain power of a prime from a fraction.□

Theorem: The trigonometric and hyperbolic functions and their inverse functions, the digamma function ψ, the Lambert-W function, the Ein function, the (hyperbolic) sine integral S(h)i, Euler's Beta function B, and, for $s, u \in {}^{\omega}\mathbb{N}^*$ and $t \in {}^{\omega}\mathbb{N}$, the generalised error function E_t, the hypergeometric function ${}_0F_t$, the Fresnel integrals C and S and the Bessel function I_t and the Bessel function of the first kind J_t, the Legendre function χ_s, the polygamma function ψ_s, the generalised Mittag-Leffler function $E_{s,t}$, the Dirichlet series $+_{n=1}^{\omega}\tilde{n}^s f(n)$ with maximally finite real $|f(n)|$, the prime zeta function $P(s)$, the polylogarithm Li_s and the Lerch zeta-function $\Phi(q, s, r)$ always yield no ω-ANs for real arguments and maximal finite real $|q|$ and $|r|$ at points where their TS converge.

Proof: GPC, Dirichlet prime number theorem and Wallis product prove the claim. For the digamma function, the claim follows from the proof of ω-transcendence of Euler's constant γ below.□

Theorem: Let $\gamma := Li_1(1) - {}_{\epsilon}\omega = \uparrow_1^{\omega} (\widetilde{[x]} - \tilde{x}) \downarrow x$ for $Li_s(z) := +_{n=1}^{\omega}\tilde{n}^s z^n$, $z \in {}^1\dot{\mathbb{C}}$ and $s \in {}^{\omega}\mathbb{C}$, where rearranging yields $\gamma \in]0, 1[$. Accepting ${}_{\epsilon}\omega = Li_1(\tilde{2})_2\omega$ shows $\gamma \notin {}^{\omega}_{\omega}\mathbb{A}_{\mathbb{R}}$ with a precision of $\mathcal{O}(\tilde{2}^{\omega}\tilde{\omega}_{\epsilon}\omega)$.

Proof: The GS implies $-_{\epsilon}(-\acute{x}) = Li_1(x) + \mathcal{O}(\tilde{\omega}x^{\dot{\omega}}/\acute{x}) + t(x)\downarrow x$ for $x \in [-1, 1 - \tilde{v}]$ and $t(x) \in {}^{\omega}\mathbb{R}$ such that $|t(x)| < \omega$. Apply Fermat's little theorem and GPC to $\mathrm{den}(\tilde{p}(1 - 2^{-p}{}_2\omega))$ for $p = \max {}^{\omega}\mathbb{P}$.□

Theorem: It holds $+_{n=-1}^{\omega}\widetilde{a_n b_n} \notin {}^{\omega}_{\omega}\mathbb{R}$ only for arbitrary $b_n \in {}^{\omega}\mathbb{N}^*$, if it does also for $+_{n=-1}^{\omega}\widetilde{a_n}$ or $\widetilde{a_{-1}} - \widetilde{a_{\omega}}$ where $a_n \in {}^{\omega}\mathbb{N}^*$, since $b_n := 1\ (+\ a_n)$ (telescope sum) may be true (cf. [889], p. 346).□

Theorem: From $p^2 \nmid \mathrm{num}(q)\mathrm{den}(q)$ for all $p \in {}^{\omega}\mathbb{P}$ where $q \in \mathrm{Q} := {}^{\omega}_{\omega}\mathbb{R}_{>0} \ni q^x$ and ${}_2\omega \gg |x| \in {}^{\omega}\mathbb{R}$, it follows that $x \in {}^{\omega}\mathbb{Z}$.

Indirect proof: Let wlog $x > 0$. Since there is no contradiction for $x \in {}^{\omega}\mathbb{N}^*$, assume $x \in Q \backslash {}^{\omega}\mathbb{N}^*$. Since this implies $q^x \in {}^{\omega}\mathrm{A}_{\mathbb{R}} \backslash \mathrm{Q}$, assume $x := k/d \in {}^{\omega}\mathbb{R}_{>0} \backslash Q$ for $d, k \in \mathbb{N}^*$ and $d \perp k$. This implies $q^k = r^d$ for an $r \in Q$. The fundamental theorem of arithmetic yields a numerator or denominator of q or r greater than 2^{ω}.□

Remark: The previous theorem proves the Alaoglu and Erdős conjecture, which states that p^x and q^x are ν-real for distinct $p, q \in {}^{\nu}\mathbb{P}$ if and only if $x \in {}^{\nu}\mathbb{Z}$ and $|x|$ is not excessively large.

Example: Every $m_0 \in {}^{(\omega)}\mathbb{N}^*$ implies $m_{\tilde{\imath}} = 1$ if there are no cycles in $m_{\grave{k}} := \left\lfloor m_k^{\frac{3-\chi_{2\mathbb{N}}(m_k)}{}} \right\rfloor$.

Collatz theorem: Every $n_0 \in {}^{(\omega)}\mathbb{N}^*$ implies $n_{\tilde{\imath}} \in \{1, 2, 4\}$ where $\hat{n}_{\grave{k}} := n_k + \chi_{2\mathbb{N}}(\acute{n}_k)(5n_k + 2)$.

Proof: If the trivial cycle (here by (Slapničar, Ivan: *There are no cycles in the 3n + 1 sequence*, arXiv: 1706.08399v1)) is the only one, every such oftener descending than ascending procedure ends up below every $n_0 \geq 2$, since the expected value is ¾ of the previous one.□

Definition: When two numbers $x_0, y_0 \in {}^{\omega}\mathbb{C}^*$ do not satisfy any non-trivial polynomial equation $p(x, y) = 0$, so they are called *ω-algebraically independent*.△

Theorem: The GPC, with $\epsilon = (1 + \tilde{p})^p$ for maximal $p \in {}^{\omega}\mathbb{P}$ and π as Wallis product, yields pairwise ω-algebraically independent representations of A, C_2, γ, ϵ, K and π.□

Remark: These conditions are not sufficient as the examples $a_n := 1, b_n := 2$ resp. $(a_n) := (12, 12, 12, 12, 12, 6, 12, 20, 30, 42, \ldots, \acute{\omega}\omega)$, $b_n := 1$ show for the sums $\breve{\omega} + 1$ resp. $(\omega - 2)/\acute{\omega}$. Considering $(n!)$ resp. (a_n) where $\acute{a}_{\grave{n}} = a_n \grave{a}_n$, it holds that $\bigplus_{n=1}^{\omega} \widetilde{a_n b_n} \notin {}^{\omega}_{\omega}\mathbb{A}_{\mathbb{R}}$ for $b_n := n + 2$ resp. $b_n := 1$.

Beal's theorem: Equation $a^m + b^n = c^k$ for $a, b, c \in \mathbb{N}^*$ and $k, m, n \in \mathbb{N}_{\geq 3}$ implies $\gcd(a, b, c) > 1$.

Proof: Exponentiation implies $\gcd(a, c) > 1$ from $b^n = (c^{kq} - a^{mr})(\tilde{c}^{k\acute{q}} + \tilde{a}^{m\acute{r}}) = c^k - a^m + c^{kq}\tilde{a}^{m\acute{r}} - \tilde{c}^{k\acute{q}} a^{mr}$, since $f(q, r) := c^{k(\hat{q}-1)} - a^{m(\hat{r}-1)} = 0$ in $q, r \in {}^{\omega}\mathbb{R}_{>0}$ is continuous not only for the solution $(q_0, r_0) = (\check{1}, \check{1})$.□

Conclusion: The Fermat-Catalan conjecture can be proven analogously and an infinite descent implies because of $\gcd(a, b, c) > 1$ that no $n \in \mathbb{N}_{\geq 3}$ satisfies $a^n + b^n = c^n$ for arbitrary $a, b, c \in \mathbb{N}^*$.□

Erdős-Moser theorem: Faulhaber's formula implies for $k, n \in {}^{\omega}\mathbb{N}^*$ that from $n\grave{n} \mid \bigplus_{m=1}^{n} m^k = \grave{n}^k$ only the solution $k = 1 = \acute{n}$ follows due to $1 < n \nmid \grave{n}$.□

Three-cube theorem: It holds $S := \{n \in \mathbb{Z} : n \neq \pm 4 \bmod 9\} = \{n \in \mathbb{Z} : n = a^3 + b^3 + c^3 + 3(a + b)c(a - b + c) = (a + c)^3 + (b - c)^3 + c^3\} \subset a^3 + b^3 + c^3 + 6\mathbb{Z}$, since independent mathematical induction by $a, b, c \in \mathbb{Z}$ first shows $\{0, \pm 1, \pm 2, \pm 3\} \subset S$, and then the claim.□

Brocard's theorem: It holds that $\{(m, n) \in {}^{\omega}\mathbb{N}^2 : n! + 1 = m^2\} = \{(5, 4), (11, 5), (71, 7)\}$.

Proof: From $n! = \acute{m}\grave{m}$, it follows that $m = \hat{r} \pm 1$ for $r \in {}^{\omega}\mathbb{N}^*$ and $n \geq 3$. Thus $n! = \hat{r}(\hat{r} \pm 2) = 8s(\hat{s} \pm 1)$ holds for $s \in {}^{\omega}\mathbb{N}^*$. Let $2^q \mid n!$ and $2^{\grave{q}} \nmid n!$ for maximal $q \in {}^{\omega}\mathbb{N}^*$. Therefore $n! = 2^q(\hat{u} + 1)$ holds for $u \in {}^{\omega}\mathbb{N}^*$ and necessarily $n! = 2^q(2^{q-2} \pm 1)$. Then the prime factorisation of $n!$ requires $n \leq 7$ giving the claim.□

Wilson's prime number theorem: Only the primes $p \in \{5, 13, 563\}$ satisfy $q := p^2 \mid (\acute{p}! + 1)$.

Indirect proof: Adding and subtracting afterwards a suitable power of two on the left-hand side, division by $\acute{p}$ or $\grave{p}$ yields in $\hat{n}q + \acute{q} = \acute{p}!$ for $n \in {}^{\omega}\mathbb{N}^*$ a higher one on the right-hand side for sufficiently big p.□

Littlewood theorem in conventional mathematics: For all $a, b \in {}^{\nu}\mathbb{R}$ and $n \in {}^{\nu}\mathbb{N}^*$, it holds that $\liminf_{n\to\infty} n\|na\|_d\|nb\|_d = 0$ where $||\cdot||_d$ is the *distance to the nearest integer*.

Proof: For $k, m \in {}^{c}\mathbb{N}^*$ as denominators of the continued fraction of a resp. b with precision $g \in {}^{\nu}\mathbb{R}_{>0}$ and n/km again and again integer, Dirichlet's approximation theorem (see [455], p. 63) yields that:

$$\liminf_{n\to\infty} n\|na\|_d\|nb\|_d = \liminf_{n\to\infty} n\,\mathcal{O}(\tilde{n})^2 = \liminf_{n\to\infty} \mathcal{O}(\tilde{n}) = 0. \square$$

Refutation by nonstandard mathematics: For $a = b := \tilde{\omega}^{\tilde{3}}$, it holds that $\omega\|\omega a\|_d\|\omega b\|_d = 1. \square$

Example: For s ∈ ${}^{(\omega)}\mathbb{C}$ where Re(s) ≤ 1 and $z := \tilde{2}^{\acute{s}}$, $\zeta(\mathrm{s}) = +_{n=1}^{\omega} \tilde{n}^s$ has definitely no analytic continuation (cf. [949], p. 4) and no zeros. This disproves the Riemann hypothesis:

$$\mp_{n=1}^{\omega} \tilde{n}^s = z +_{n=1}^{\breve{\omega}} \tilde{n}^s - +_{n=1}^{\omega} \tilde{n}^s \neq \acute{z} +_{n=1}^{\omega(/2)} \tilde{n}^s.$$

Theorem: Since the Dirichlet L-function $L(s,\chi) = +_{n=1}^{\omega} \chi(n)\tilde{n}^s$, has only zeros for s = 0 and nontrivial Dirichlet characters $\chi(n)$, it disproves the generalised Riemann hypothesis.□

Octal Computation of Time

We measure time so far in years, months, weeks, days, hours, minutes, and seconds. Seconds can be divided into respectively thousandths, step by step more finely down to the measuring border. But is our computation of time practical?

The following three questions are to clarify the problems:

1. How many days are between 25.01.2004 and 11.11.2004?
2. Which weekday is the 11.11.2004?
3. How many seconds are between 09:12: 06 o'clock and 11.11.11 o'clock?

The answer to question 1 has to consider the different length of the months and leap years.

The answer to question 2 in addition has to include the seven day rhythm of the week and to assume a date with well-known weekday.

The answer to question 3 requires the conversion of hours and minutes in 3600 or rather 60 seconds.

Hence, I suggest the following computation of time:

1. The octal system is used throughout.
2. Time is specified: [[+/-] year,][day].[fraction of the day][additive].
3. An Earthday is inserted between Sunday and Monday within the week.
4. The months have regularly four eight-day weeks.
5. The last twelfth month has deviating 13 days (in the leap year 14).
6. Day 0 is New Year (1.1. so far, Sunday), day 1 is working day (Earthday).
7. The year will be introduced with year 0.
8. Time zones are omitted and Greenwich Mean Time is used world-wide.

Explanations:

To 1. (The octal system is used throughout.):

We mostly use the decimal system (radix 10, figures from 0 to 9). The radix indicates the number of different figures of a number system. Numbers can be equivalently represented in other number systems (only with another "appearance").

Numbers that are larger than indicated by the highest figure transfer a carry of one to the next left place: After the 9 follows the 10, after the 99 the 100, after the 999 the 1000 etc.

Humans can on the average recognise maximally seven things at one time, without having to count them. The octal system with the radix 8 possesses the figures from 0 to 7 and accommodates here thus the nature of humans.

The octal system can be converted in a simple manner into other number systems that are a power of the radix two. Apart from the binary or dual system (radix 2, figures 0 and 1), there are among these in particular the tetral system (radix 4, figures from 0 to 3) and the hexadecimal system (radix 16, figures from 0 to 9 and from A to F).

Most computers are based on the binary system. The figures 0 and 1 indicate whether current flows or not. Numbers in the decimal system have to be converted for these computers into the binary system so that it can be counted fast.

The longer the numbers are, the more time-consuming the conversion is. For the output in the decimal system, a renewed transformation is necessary. By the conversion between dual and decimal system, frequently transformation-caused truncations arise behind the decimal point.

Here some examples (the subscript number indicates the radix for the distinction of the number systems):

$2004_{10} = 7D4_{16} = 3724_8 = 133110_4 = 11111010100_2$.

If 11111010100_2 starting from right is divided by | into two-blocks in the form $1|11|11|01|01|00_2$, then the representation for 133110_4 can be obtained by the following correspondences:
00 ≙ 0, 01 ≙ 1, 10 ≙ 2, and 11 ≙ 3. A first block with one figure remains unchanged.

To obtain 3724_8 from 11111010100_2, three-blocks from right have to be transformed to: $11|111|010|100_2$. The correspondences read:
000 ≙ 0, 001 ≙ 1, 010 ≙ 2, 011 ≙ 3, 100 ≙ 4, 101 ≙ 5, 110 ≙ 6, and 111 ≙ 7. A first block with less than three figures has to be completed before by zeros from the left.

To obtain $7D4_{16}$ from 11111010100_2, blocks of four from right have to be transformed to: $111|1101|0100_2$. The correspondences read:
0000 ≙ 0, 0001 ≙ 1, 0010 ≙ 2, 0011 ≙ 3, 0100 ≙ 4, 0101 ≙ 5, 0110 ≙ 6, 0111 ≙ 7, 1000 ≙ 8, 1001 ≙ 9, 1010 ≙ A, 1011 ≙ B, 1100 ≙ C, 1101 ≙ D, 1110 ≙ E, and 1111 ≙ F. A first block with less than four figures has to be completed before by zeros from the left.

Generally, if you have a radix of 2^n, there have to be formed blocks of n figures in the representation of the dual system from the right and the first block to the left has to be filled up with zeros up to n figures, beginning from the left. The transformation functions also in reverse from the 2^n-system into the binary system. Furthermore, there are still different algorithms for the transformation.

To obtain $7D4_{16}$ from 2004_{10}, 2004_{10} is continually divided by 16_{10}:
$2004_{10}/16_{10} = 125_{10}$ remainder of 4_{10};
$125_{10}/16_{10} = 7_{10}$ remainder of 13_{10};
$13_{10}/16_{10} = 0_{10}$ remainder of 7_{10}.

The two digit remainders are converted into the hexadecimal system and gone through from bottom to top. The desired result $7D4_{16}$ is obtained if they are thereby lined up from left to right: $7_{10} = 7_{16}$,$13_{10} = D_{16}$ and $4_{10} = 4_{16}$.

Generally, the number to be converted has to be continually divided by the radix of the target system, and the multi-digit remainders have to be converted into the notation of the target system. The number searched for results from the lining up of the remainders from left to right if in the division calculation the remainders are gone through from bottom to top.

Naturally, here different transformation algorithms exist - also in the reverse direction. Since the continual division is computationally more expensive than the figure replacement in figure blocks, the mentioned loss of time emerges. Positions behind the floating point are similarly converted.

To 2. (Time is specified: [[+/-] year,][day].[fraction of the day][additive].):

The order results from the sorting capability of the date. + can be omitted; - refers only to the year. If the fraction of the day is omitted, the (normal) date results. The year can be also omitted if the understanding permits that.

The fraction is indicated as sequence of octal figures. For optical reasons, the numbers can be filled up with zeros on the right to an even number (e.g. four or six-digit). Additive may be for example: Christian, Jewish or Islamic or an abbreviation. The standard is without additive.

The seasons spring, summer, autumn and winter and the holidays return regularly. These regularities shall survive with respect to the year, even if there are irregularities in the details.

Hours are not used anymore for reasons of confusion. The day is divided roughly into eight parts (called octs). Furthermore, an oct is divided into eight parts (called lepts). The minute is redefined as the 64th part of a lept. The second is redefined as the 262144th part of a day (64 * 64 * 64 = 262144).

For this reason, time can be indicated approximately three times more exactly with six figures in the octal system as with six figures in the decimal system (262144/[24 * 60 * 60] ~ 3.034). Four lepts, or a half oct,

correspond to 1.5 hours and two lepts correspond to a school hour of 45 (old) minutes. The break of five minutes becomes the break of a quarter-lept.

Because $64_{10} = 100_8$, seconds pass naturally into minutes, minutes likewise into lepts and lepts into days. Days cannot pass naturally into years because a normal year has $365_{10} = 555_8$ days. The leap year with $366_{10} = 556_8$ days does not improve anything here.

The new analogue clock dial is much more symmetrically designed than the old one and corresponds better to the compass rose and the sundial. x and + are superposed as principal axes. The 0 is placed at the bottom. The digits follow clockwise from 1 to 7. The division into 64 lines (lepts, new minutes and seconds) optically hardly differs from the conventional one into 60 lines (old minutes and seconds).

To 3. (An Earthday is inserted between Sunday and Monday within the week.):

The weekdays are named after planets in several languages. The Earth was excluded so far. It is appreciated by the Earthday. Finally, it is less crucial, which name a language community uses.

A week with eight days represents at first sight a greater human burden. Wednesday could, however, bring a relief as middle of the week if the afternoon is kept as free as possible, both on the job and at school.

We assume an eight-hour working day and two free weekend days. If we consider 56 days at those 320 hours would have been worked in the seven-day week, 336 hours in the eight-day week, then it results for Wednesday that 16/7 hours per eight-day week would have to be less worked.

If we would begin our work in GMT at 7 o'clock, then we worked on an eight-hour working day till 15:30 o'clock if we assume a half hour of lunch time. On a Wednesday would have to be worked at same beginning time only till approx. 12:43 o'clock.

We are free to work longer at flexible work time the week over in order to be able earlier to terminate our work on Friday for example. Modified solutions are to be used respectively at other work time models and work times.

To 4. (The months have regularly four eight-day weeks.):

Having 32 days, the months can be halved very well by steps of two. The month can be determined in the octal system from the three digits representing the day - where necessary zeros may be completed on the left. 0 - 3 in the second place characterise odd, 4 - 7 even months.

Month is, however, only a transient term and finally derived from the Moon, which has subordinated importance in the new computation of time. Instead the terms half, third, quarter, sixth and twelfth (of the year) are used. The date is to be indicated, if possible, accurately to week or day.

To 5. (The last twelfth month has deviating 13 days (in the leap year 14).):
The position of the leap day in the middle of the year was so far little plausible. At the end of the year, the computation of time can be corrected expediently as missing time units are inserted - in particular in seconds. To New Year, all clocks are then reset to [year],0.000000. Here fields will profit where special accuracy is needed.

The last five (in the leap year six) days can receive own weekday names. The weekday names will become less important, however, in favour of the numbers connected with them.

To 6. (Day 0 is New Year (1.1. so far, Sunday), day 1 is working day (Earthday).):

It has practical reasons that each month begins with Sunday. The beginning of the week with Sunday has historical reasons and is also a concession to the moon calendar. In addition, the 0 resembles the symbol for sun (sun calendar).

Since the Gregorian calendar at present is the most common calendar, New Year is the 1.1. according to it also in the octal calendar. The celebrations around the turn of the year can be committed in well-tried form.

To 7. (The year will be introduced with year 0.):

This happens for practical reasons: The year can be noted briefly (without additive). A clear cut is made concerning the old computation of time. This point is among all of them the least mandatory.

To 8. (Time zones are omitted and world-wide Greenwich Mean Time is used.):

The question, what time in another time zone is valid, is unnecessary. A world-wide communication is facilitated, since all clocks on Earth show the same time if they are synchronised. Greenwich historically has the zero meridian.

Now, instead of the three initial questions, the substantially more difficult one is asked:

How many days are between 25.01.2004 09:12:06 o'clock and 11.11.2004 11:11:11 o'clock?

First, the question is reformulated into the octal system (with truncation at the seconds, chr. stands for Christian):

How many days are between 3724,30.304233 chr. and 3724,473.356511 chr.?

The answer is given by a simple subtraction: 443.052256. The 25.01.2004 would be a Sunday, the 11.11.2004 would be a Tuesday in the new computation of time. There are exactly 443052256 (new) seconds between the two numerical data, rounded 4430523 (new) minutes, 44305 lepts, 4431 octs, 443 days and 44 weeks.

Discussion of important aspects:

The biggest challenge might be - independently from the computation of time - the conversion into the octal system. It is connected with high costs, great need for discussion and preparation for many years. The policy will have to work hard to convince the majority by extensive advertising efforts.

One additional day per week is for all religious communities equal (-ly fair). In relation to other week lengths, the one extended to eight days is certainly most moderate. The representation of the year is not religiously founded. By the additive, every religious community can specify its calculation of the year.

I do not doubt that humans can learn. Also handling computers was so far successfully mastered. The gap between humans and computer is strongly reduced by introducing the octal system.

Monthly sums of amount can be easily split to weekly and daily ones. Here, the shortened twelfth month can be better calculated. Equal month lengths, under normal conditions, are fairer regarding the money than the irregular ones.

Everyone not being born at a leap day celebrates the own birthday at the same weekday in a year. This is not a crucial disadvantage. The emotional rhythm of 28 days within the biorhythm is also better mixed.

The SI-unit second would have to be redefined. Here, atomic clocks should have no problem. Introducing the octal system requires to replace terms such as kilo and million respectively to shift the meaning of word beginnings such as mega, micro and nano.

Depending upon the old time zone, the day turns out to be differently related to time. Humans can begin the day, eat, work or learn, meet and sleep as they usually do. Only the time indicated differs.

We need to get used to that including the date line. It remains, however, at its old place. World-wide communication is to be given priority because of its increasing importance. The arising costs are exceeded over years by the saved ones. Calculation and communication become clearly easier.

By introducing the octal system, not everything is done. It should be connected meaningfully with a worldwide source language (as native language), also based on the octal system. Such an efficient planned language is presented below linguistics. Everyone shall be able to talk to everyone on the same level.

Synoptic octal calendar	January, February, March, April, May, June, July, August, September, October, November, *December* ***(Leap day)***			
Weekday	1st Week	2nd Week	3rd Week	4th Week
Sunday	*0th Day*	*10th Day*	20th Day	30th Day
Earthday	*1st Day*	*11th Day*	21st Day	31st Day
Monday	*2nd Day*	*12th Day*	22nd Day	32nd Day
Tuesday	*3rd Day*	*13th Day*	23rd Day	33rd Day
Wednesday	*4th Day*	*14th Day*	24th Day	34th Day
Thursday	*5th Day*	***15th Day***	25th Day	35th Day
Friday	*6th Day*	16th Day	26th Day	36th Day
Saturday	*7th Day*	17th Day	27th Day	37th Day

Further thoughts: Calculating lengths in the octal system

We measure lengths up to now on the basis of the SI-unit metre. In the octal system, there are to form appropriate multiples of 8. What is the practical implementation?

Instead of conventional kilometres of 1000_{10} m length, octal kilometres of $512_{10} = 1000_8$ m length are used. Since time is measured instead of conventional seconds in octal seconds of day length/$1000000_8 \sim 0.33_{10}$ s, it follows that almost three octal seconds are counted per conventional second.

On the tachometer, speed is measured in octal kilometre/lept, where lepts are the 100_8th = 64_{10}th part of the day. 100 kilometres per hour correspond to approximately $73242_{10} \sim 111.2_8$ lepts kilometres (octal). The difference in the numerical value may hardly be noticed and is therefore justified. 100_8 lepts kilometres (octal) correspond vice versa to approximately 87.38_{10} kilometres per hour.

The speed of light is 299792458_{10} m/s = 98808549.39_{10} m/octal second ~ 570731345_8 m/octal second. If the speed of light is specified by about 300000 km/s, we can also specify it by 570000 octal kilometres/octal second.

The 25_{10}-metre lanes are shortened to $24_{10} = 30_8$ m, 50_{10}-metre swimming to $48_{10} = 60_8$ m. Instead of 50_{10} m, 100_{10} m, 200_{10} m, 400_{10} m, 800_{10} m and 1500_{10} m would be to swim: $48_{10} = 60_8$ m, $96_{10} = 140_8$ m, $192_{10} = 300_8$ m, $384_{10} = 600_8$ m, $768_{10} = 1400_8$ m, and $1536_{10} = 3000_8$ m. These deviations are justifiable. The long hauls should be multiples of 1000_8.

In athletics, there are also the following distances: 3000_{10} m, 5000_{10} m, 10000_{10} m, 20000_{10} m and 50000_{10} m, which could be replaced by the following distances: $3072_{10} = 6000_8$ m, $4096_{10} = 10000_8$ m, $8192_{10} = 20000_8$ m, $20480_{10} = 50000_8$ m, and $49152_{10} = 140000_8$ m. The marathon will remain unchanged (42.195 km = 122.323_8 octal kilometres).

In the building industry, measures are valid that are usually exactly determined to the centimetre. Since we can accurately calculate in the octal system to $1/64_{10}$ and $1/512_{10}$ m, there should be no problems. The same is true for furniture, although it certainly can be accepted to shift measures that are multiples of 0.1 m to those that are multiples of 0.125 m.

Music

It's life that demands

by Boris Haase

Maestoso ♩ = 90

It's life that de - mandsto care still in the right time the death and its e- thics to earn some-thing just sub-lime. All

gods have in com - mon the du - ty to hold what e - ver may be the as - si - zes so bold. They

pun - ish with love all the mur - der and crime; the sieves have to fil - ter all crea - tures in time. A -

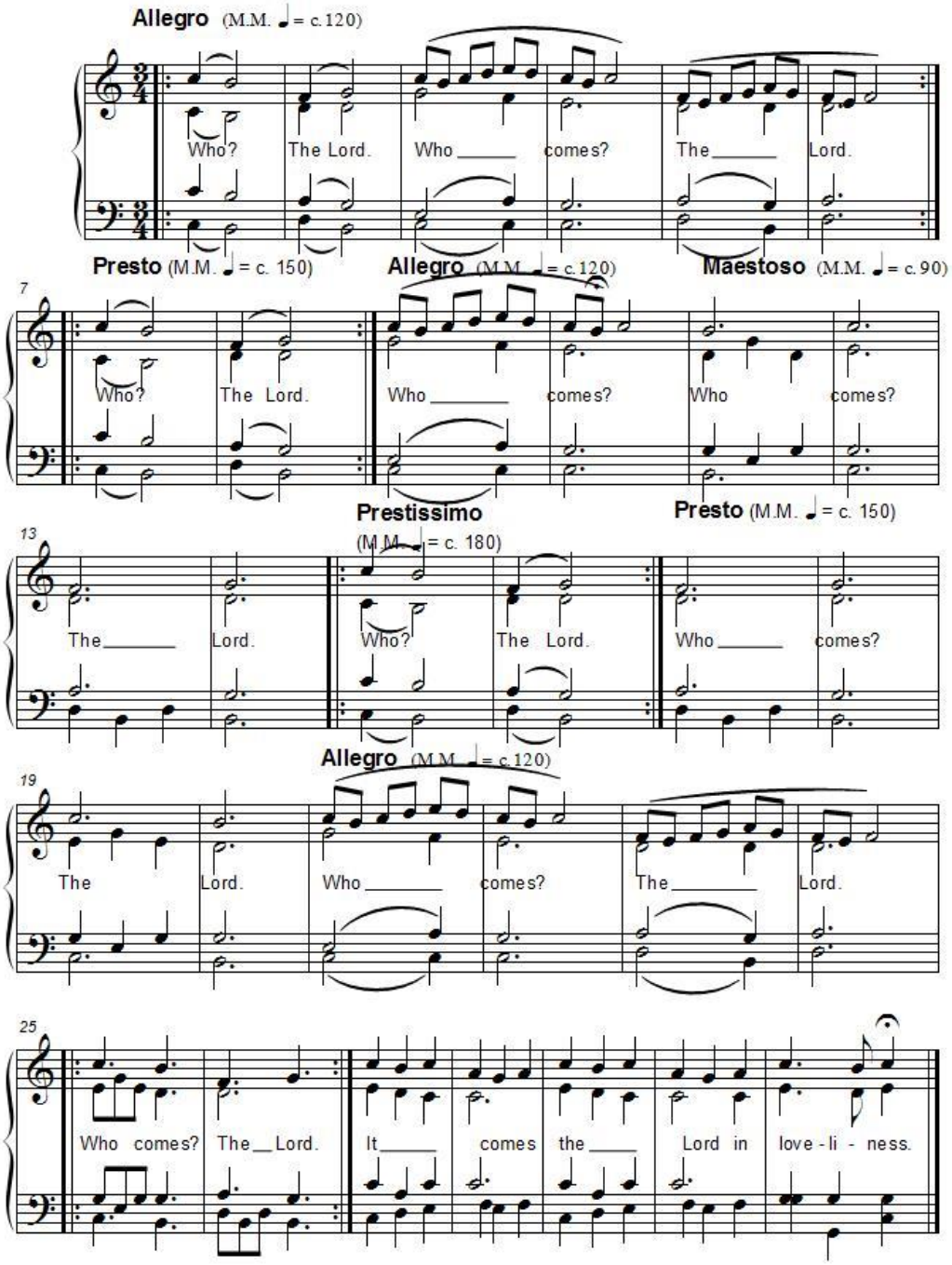
void thus the ho - ors re - mai - ning as an - chor to add inthese rounds still a plen - ty strong ran - cour. Ad-

here to the pru - dent and splen - did ad - vice: con - si - der your ac - tions in doubt bet - ter twice.

© Boris Haase

The Lord

by Boris Haase



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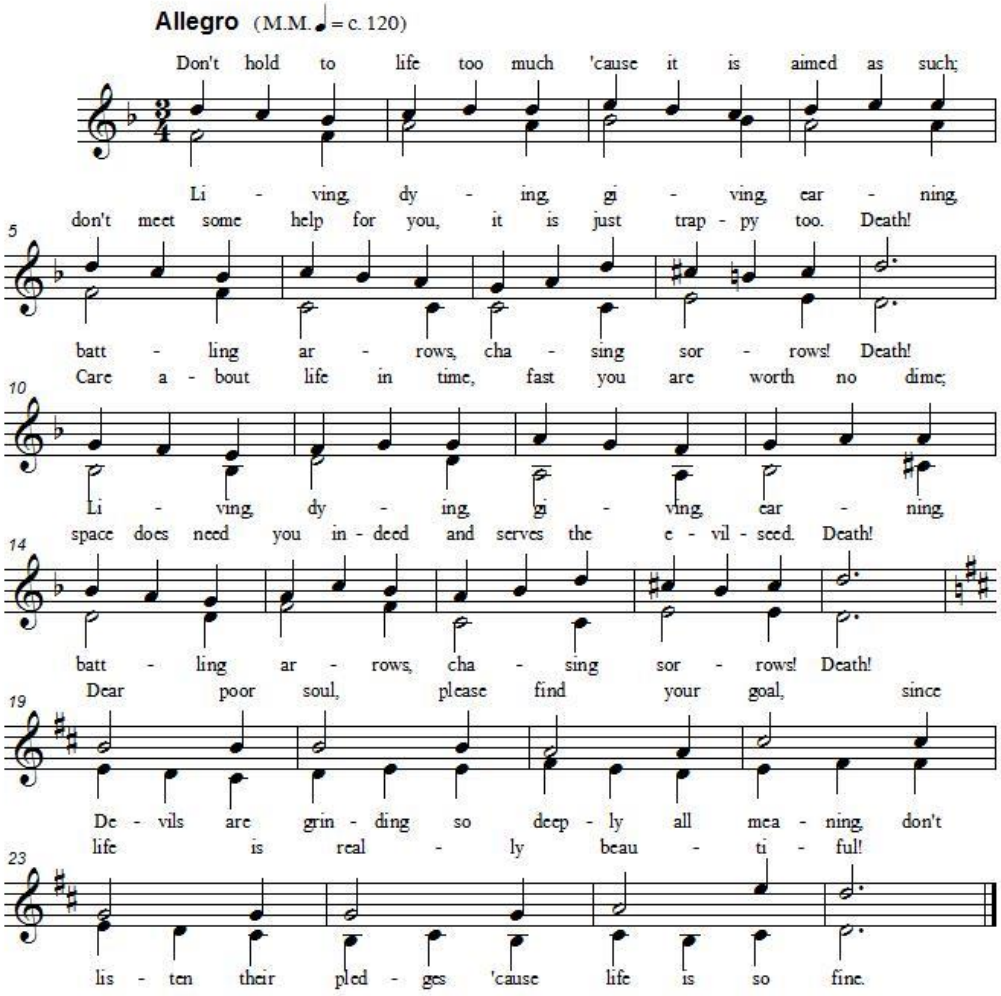
Don't hold to life too much

by Boris Haase



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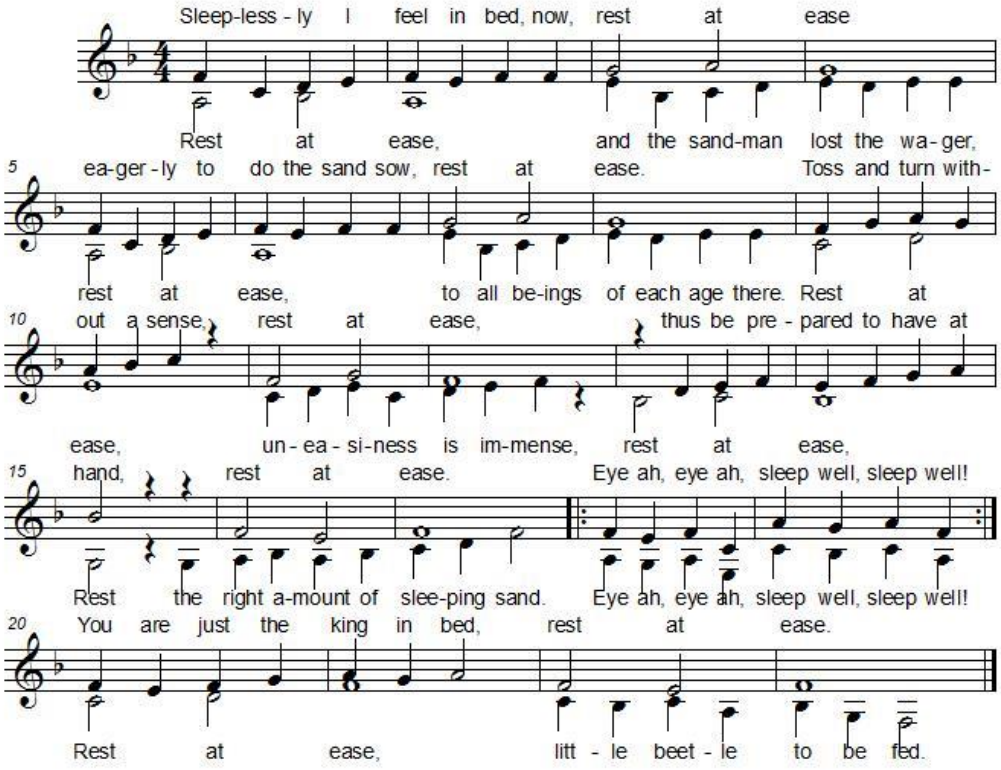
Sleeplessly I feel in bed, now

by Boris Haase



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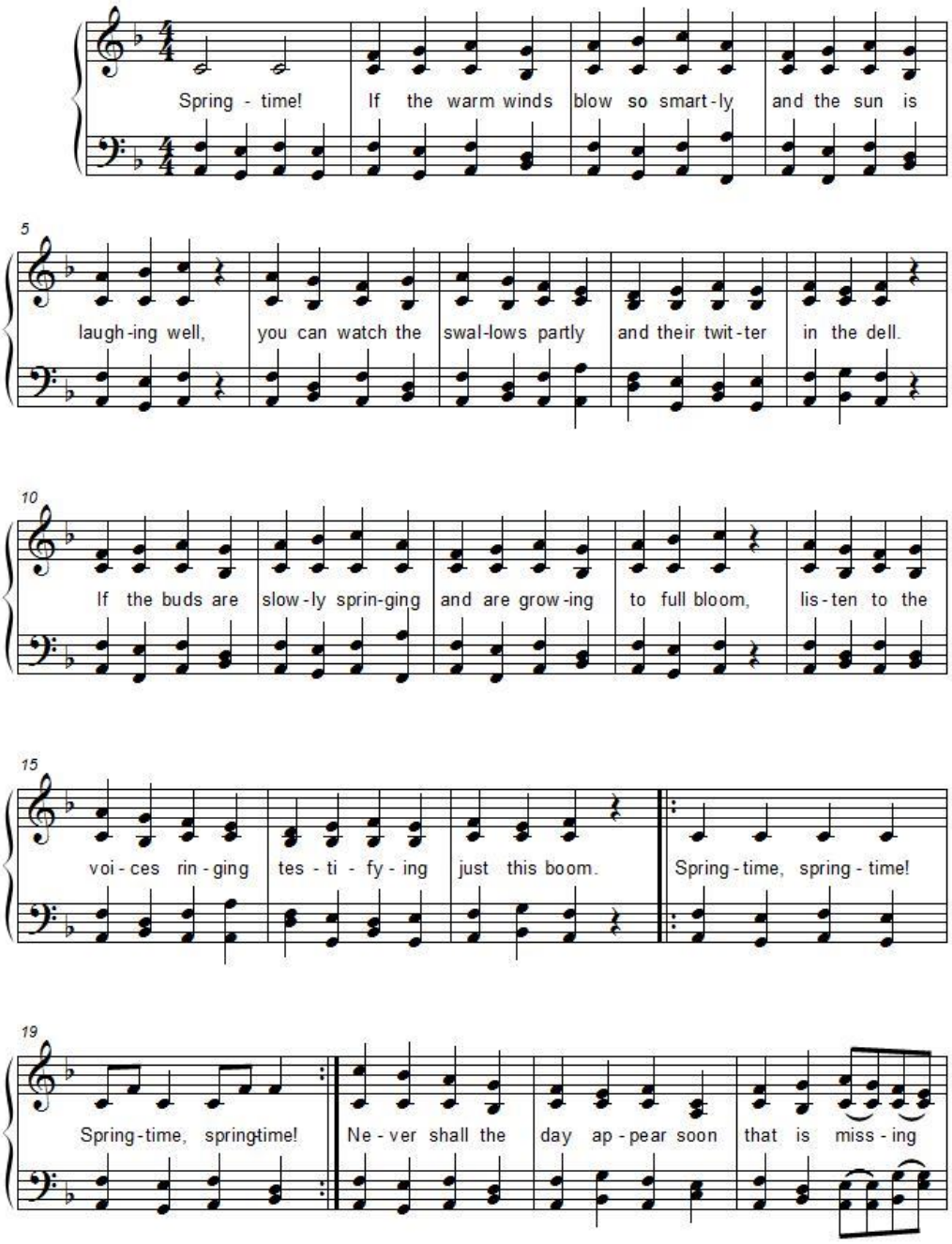
Springtime

by Boris Haase



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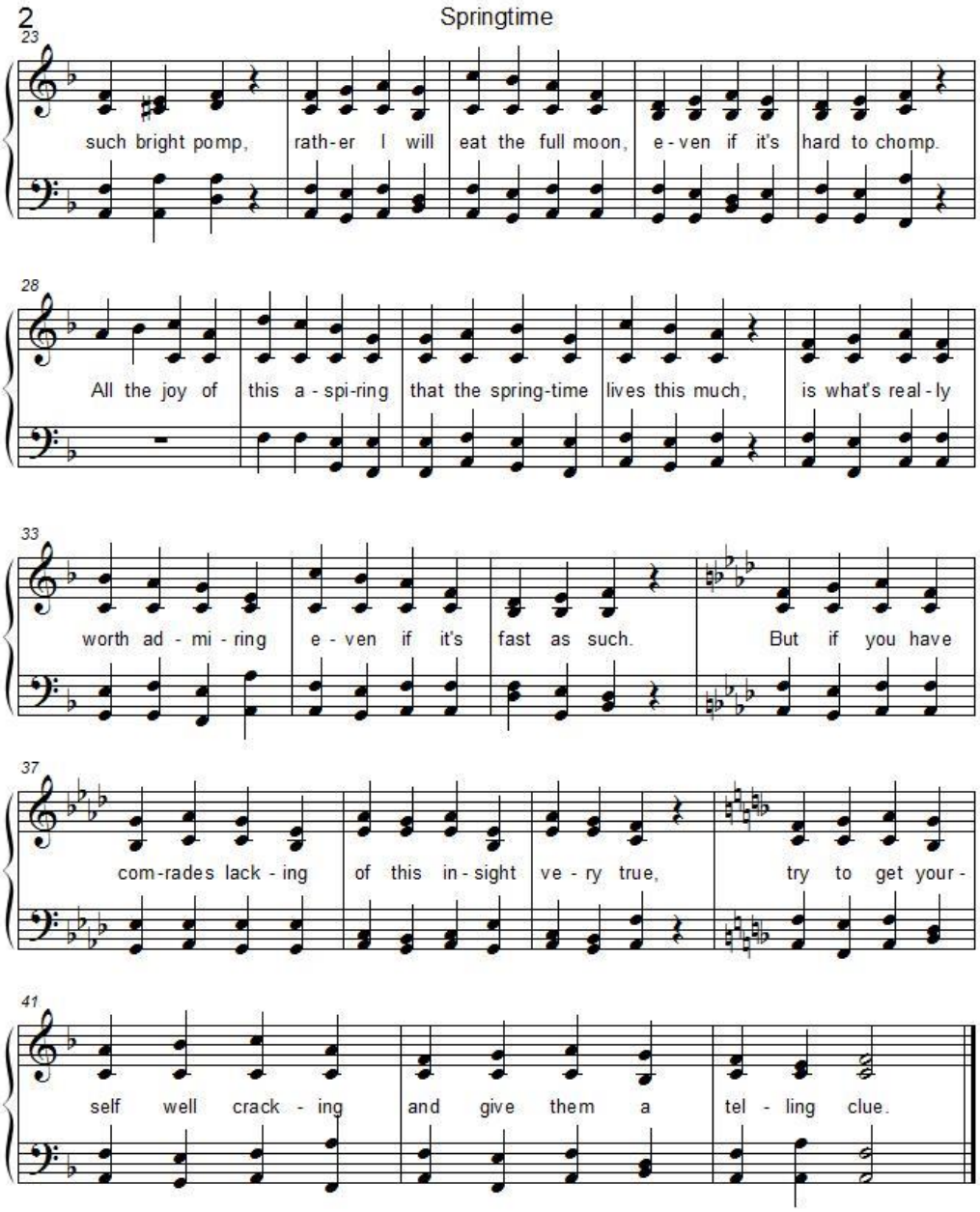
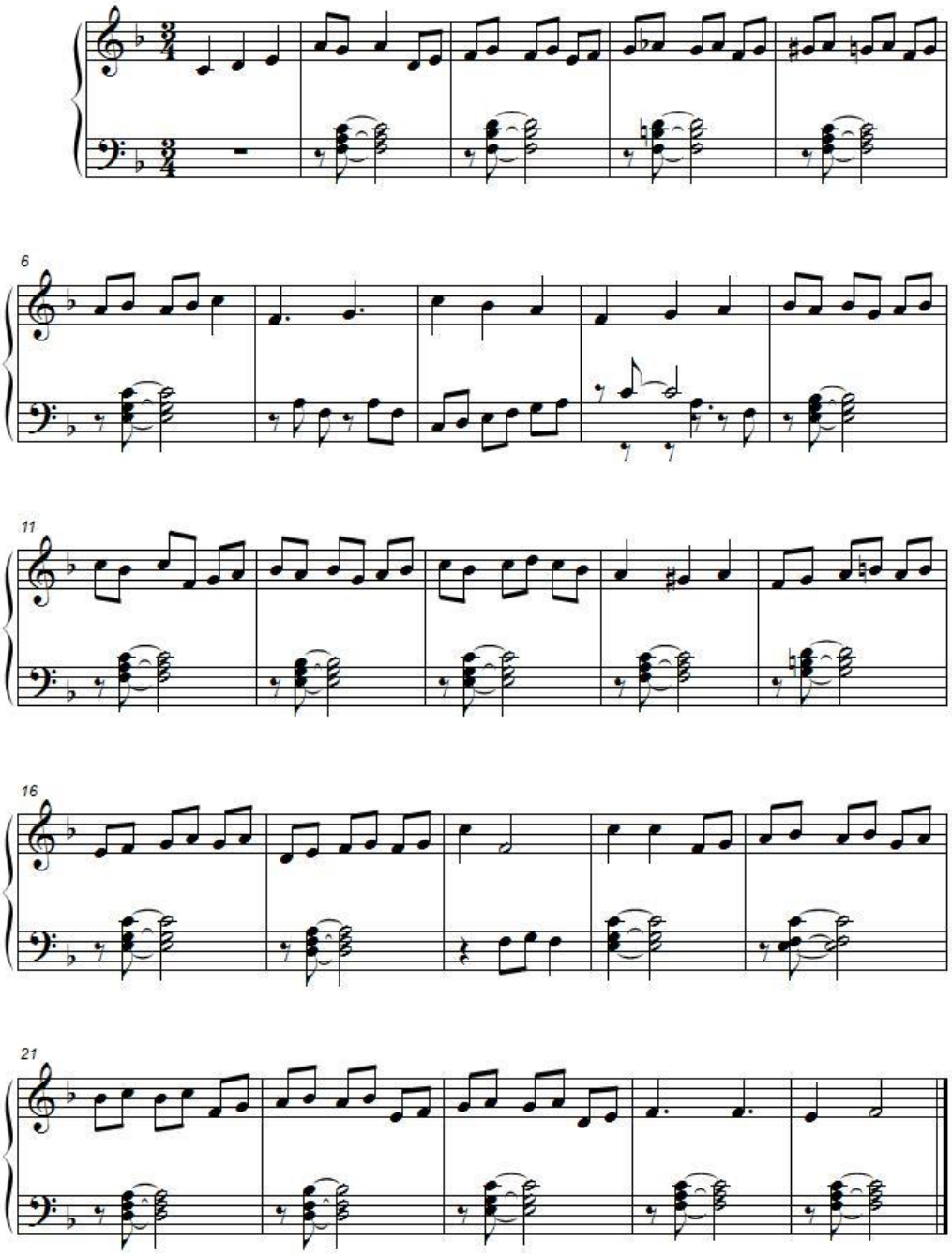

2
Springtime
23
such bright pomp, rath-er I will eat the full moon, e-ven if it's hard to chomp.
28
All the joy of this a-spi-ring that the spring-time lives this much, is what's real-ly
33
worth ad-mi-ring e-ven if it's fast as such. But if you have
37
com-rades lack-ing of this in-sight ve-ry true, try to get your-
41
self well crack-ing and give them a tel-ling clue.

Ballet exercise

by Boris Haase

Moderato (♩= c. 108)



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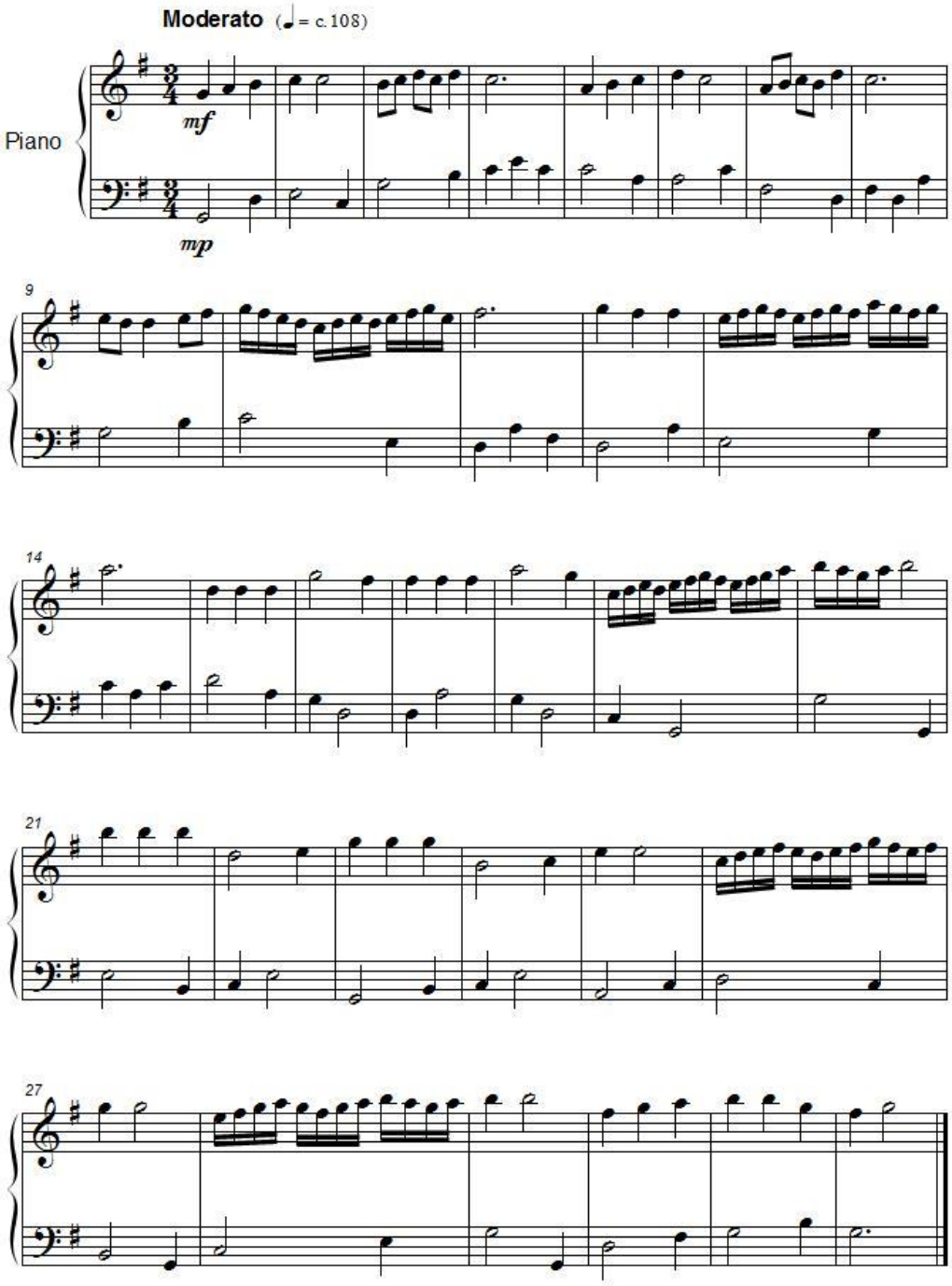
In Venice

by Boris Haase



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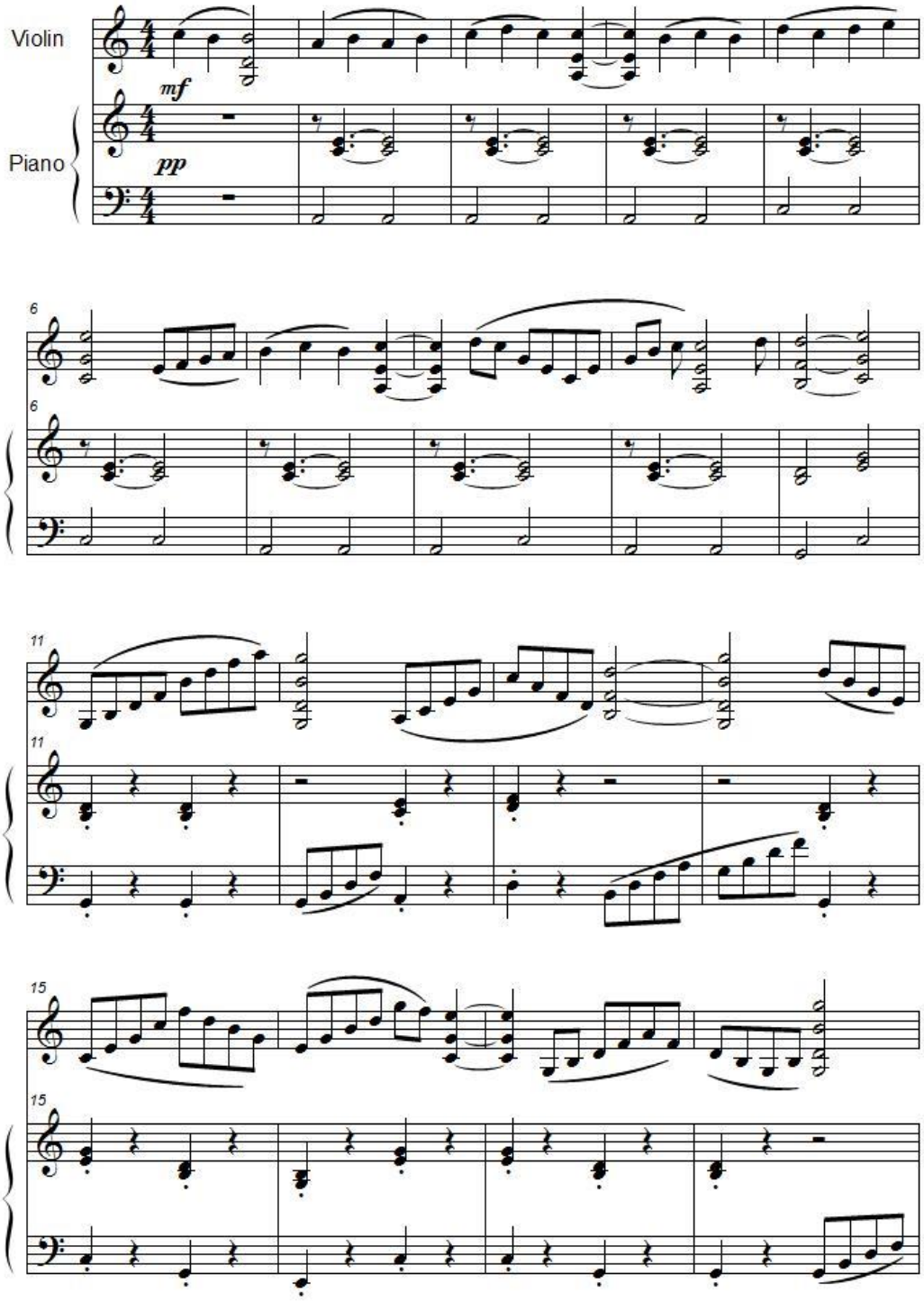
Under the bridge

by Boris Haase



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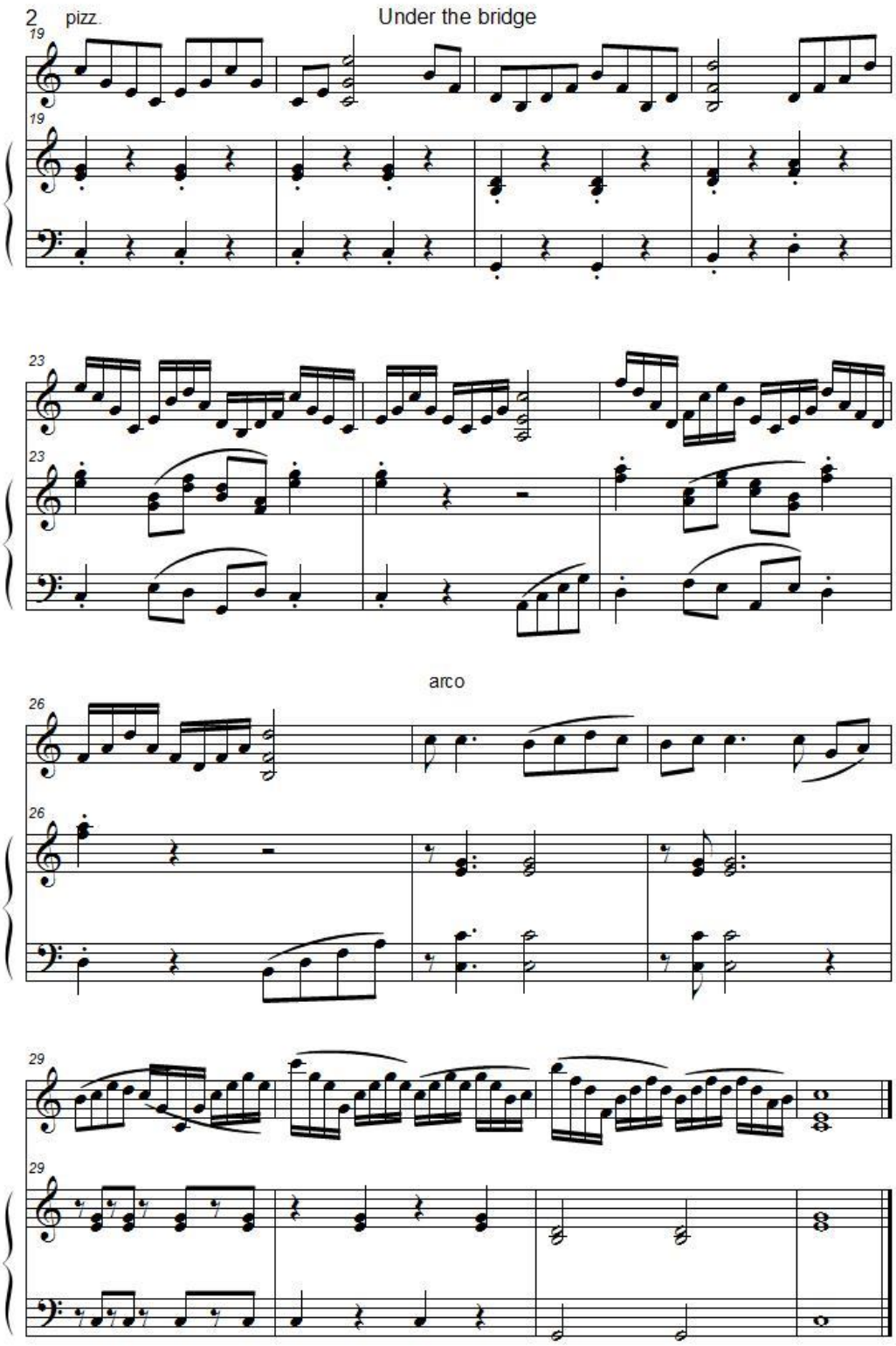
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pizz.
Under the bridge
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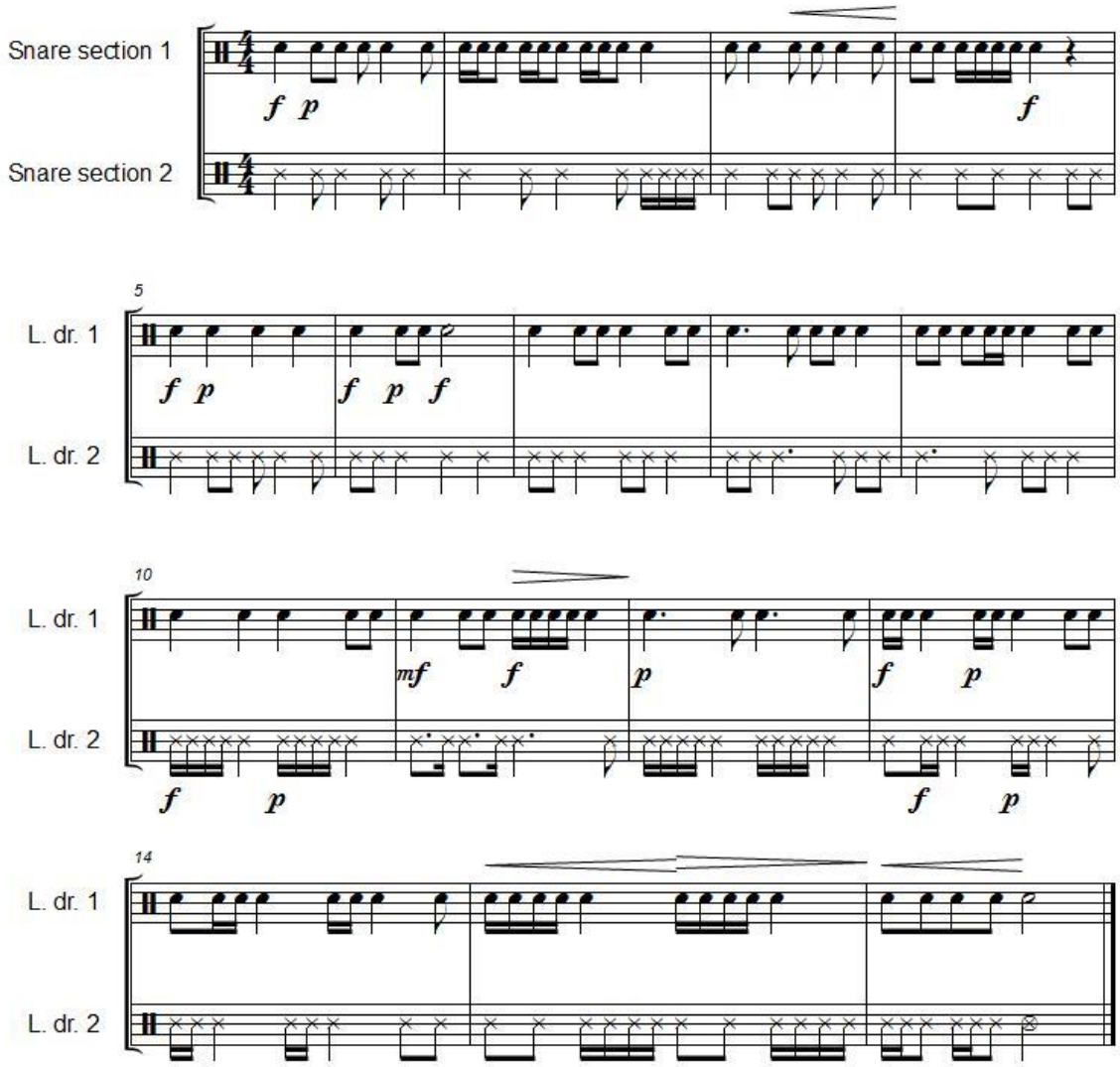
Duet of drum rabbits

by Boris Haase



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At the fair

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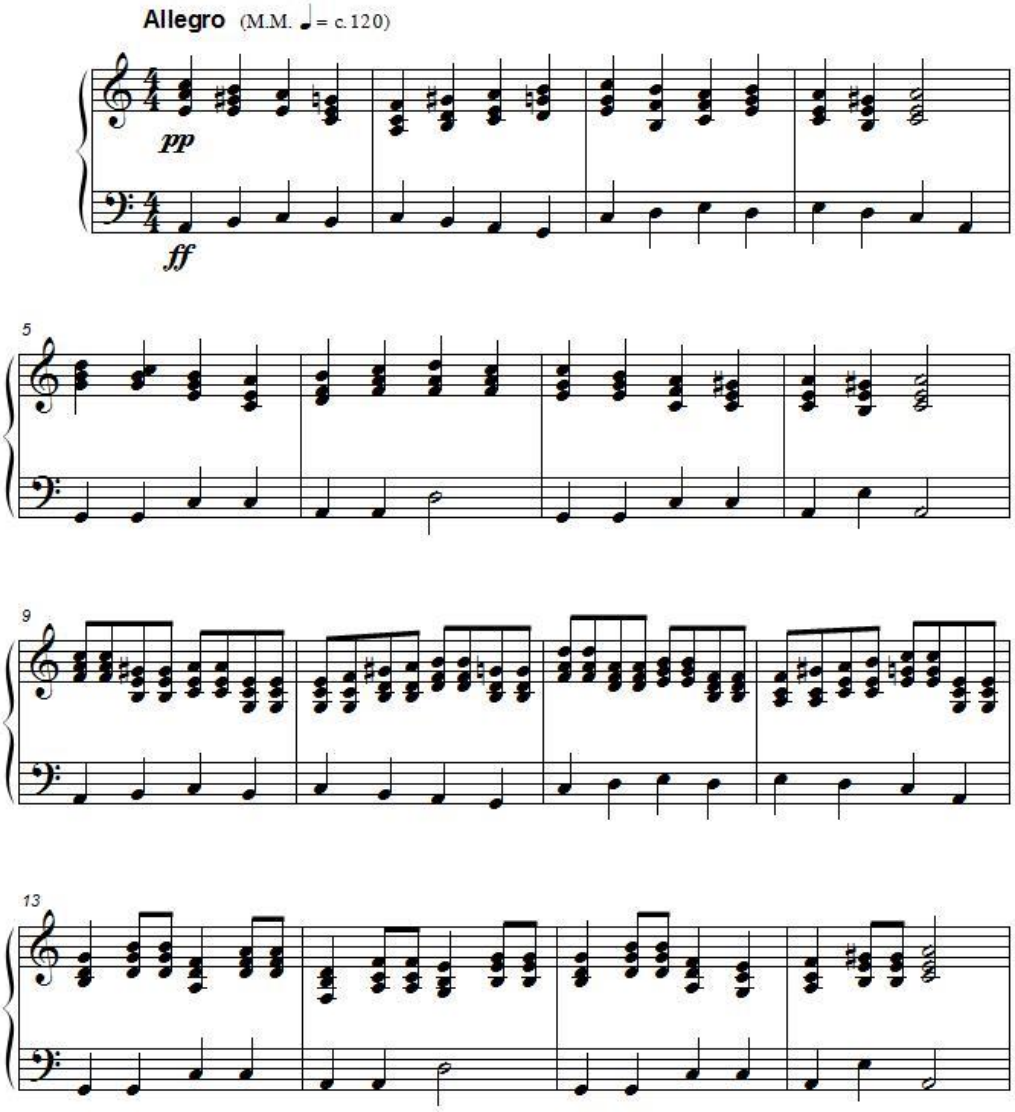

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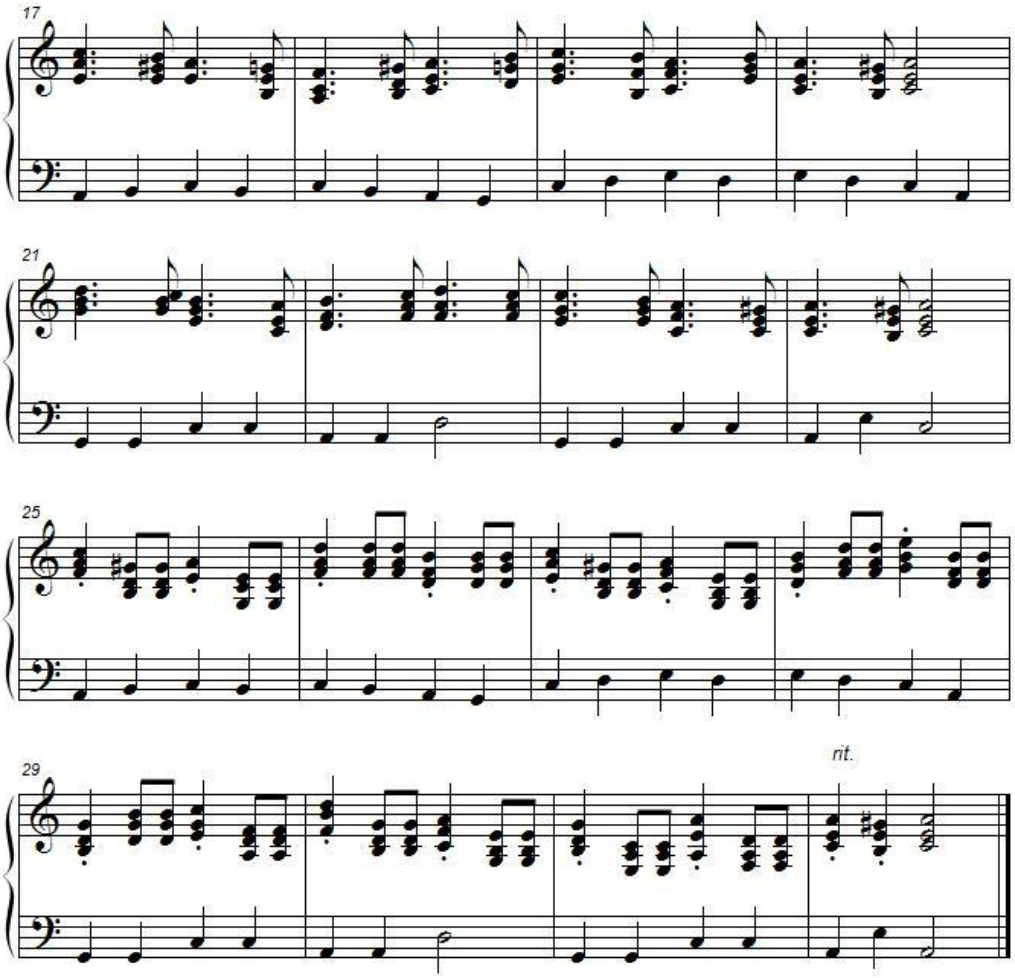
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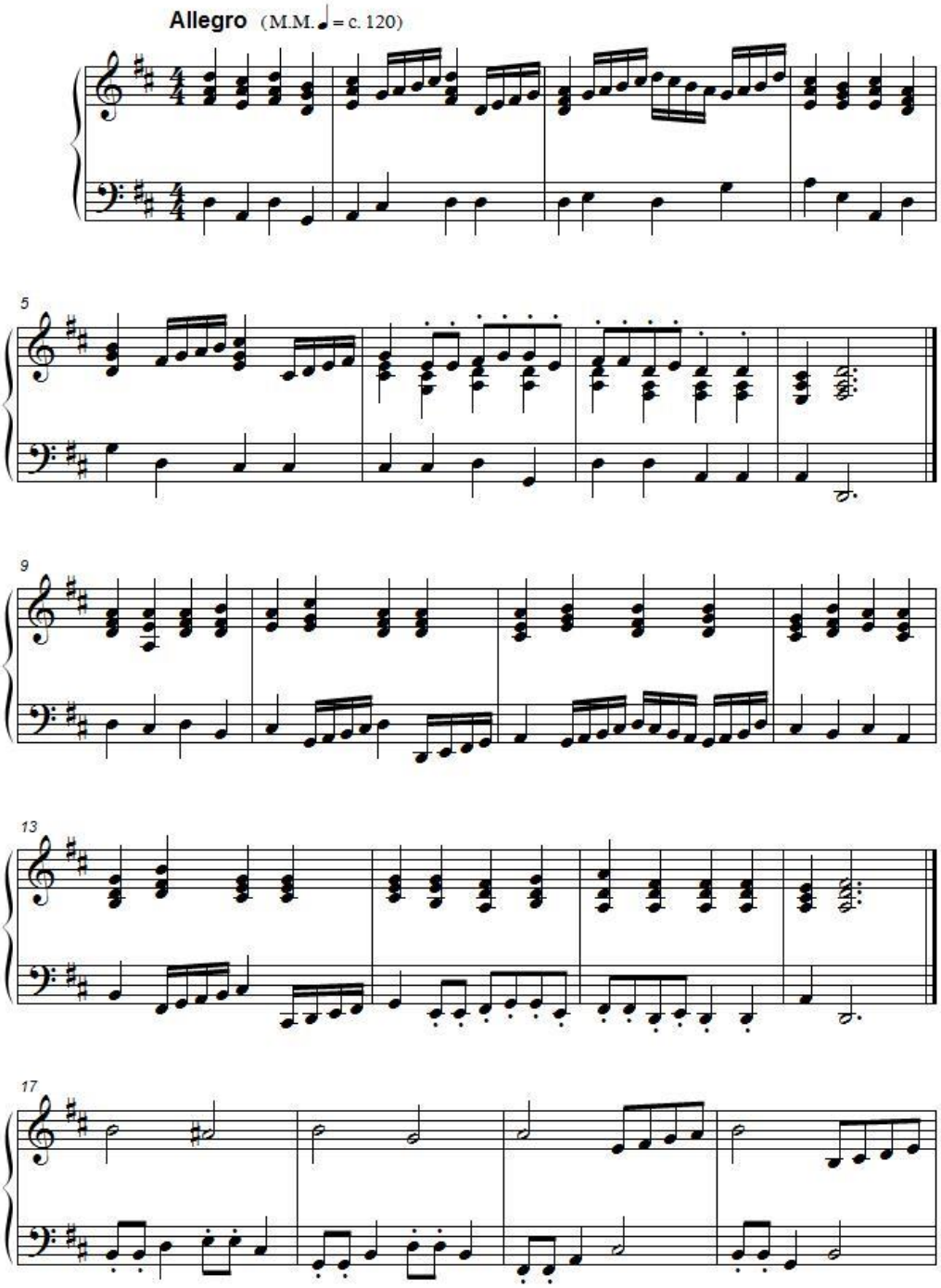
Bellflowers

by Boris Haase



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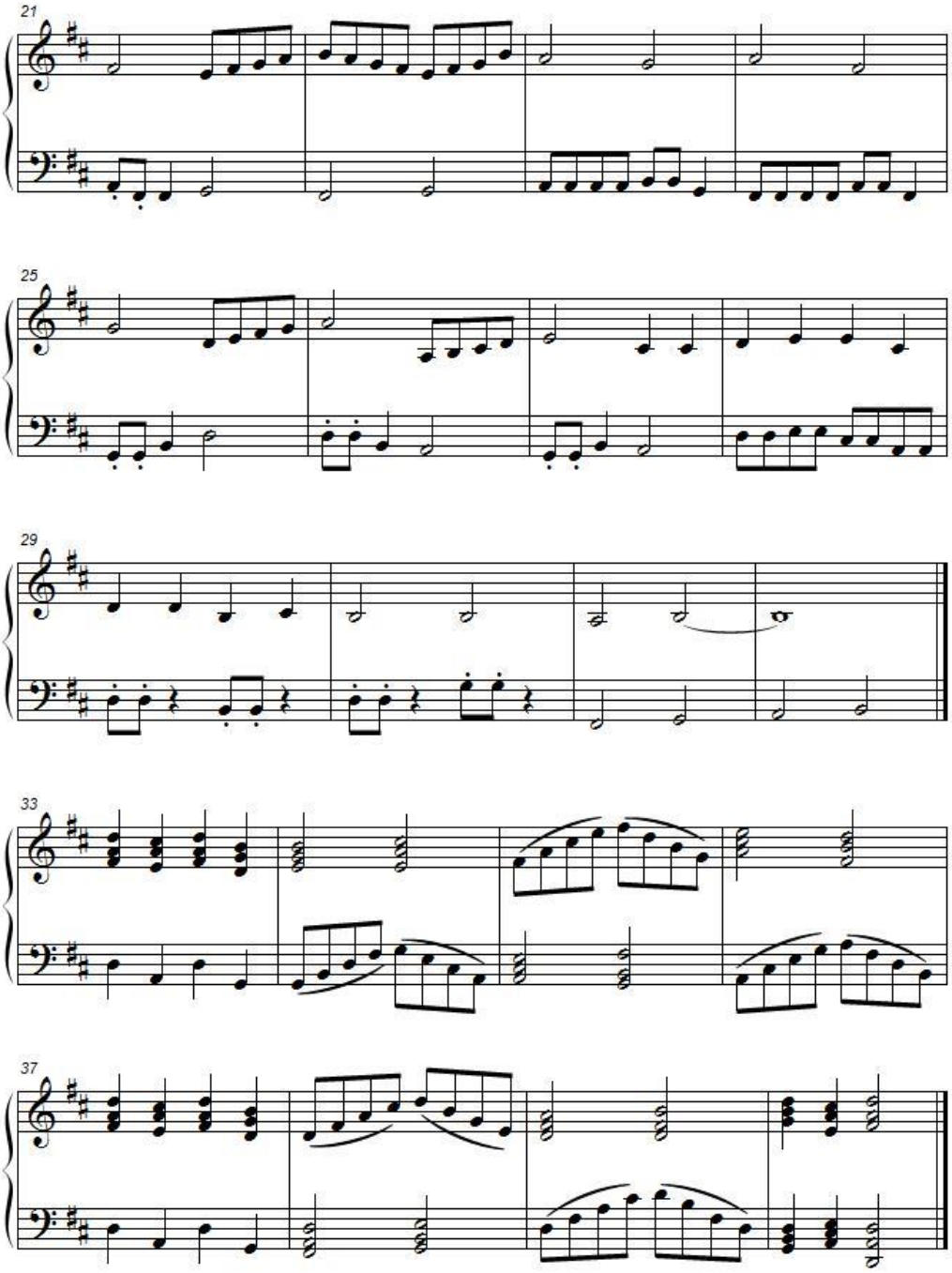
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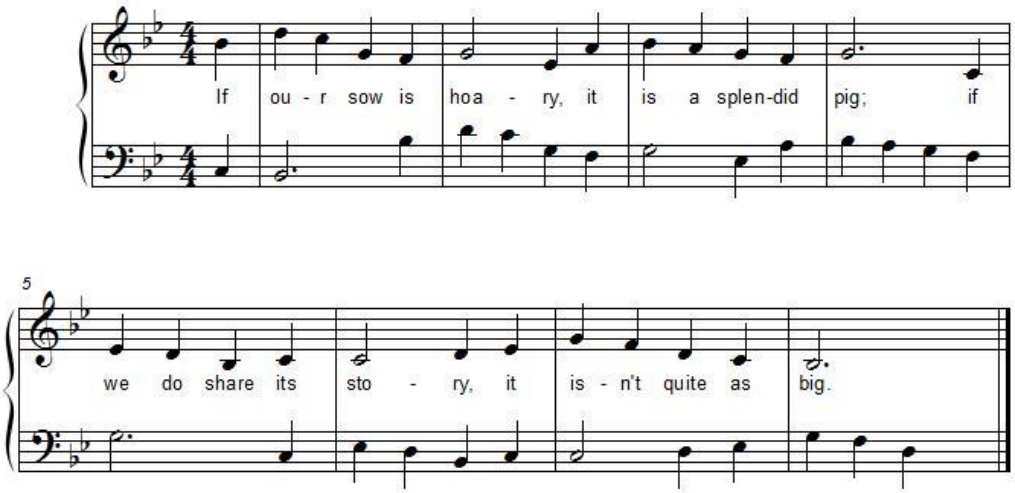
The sow

by Boris Haase



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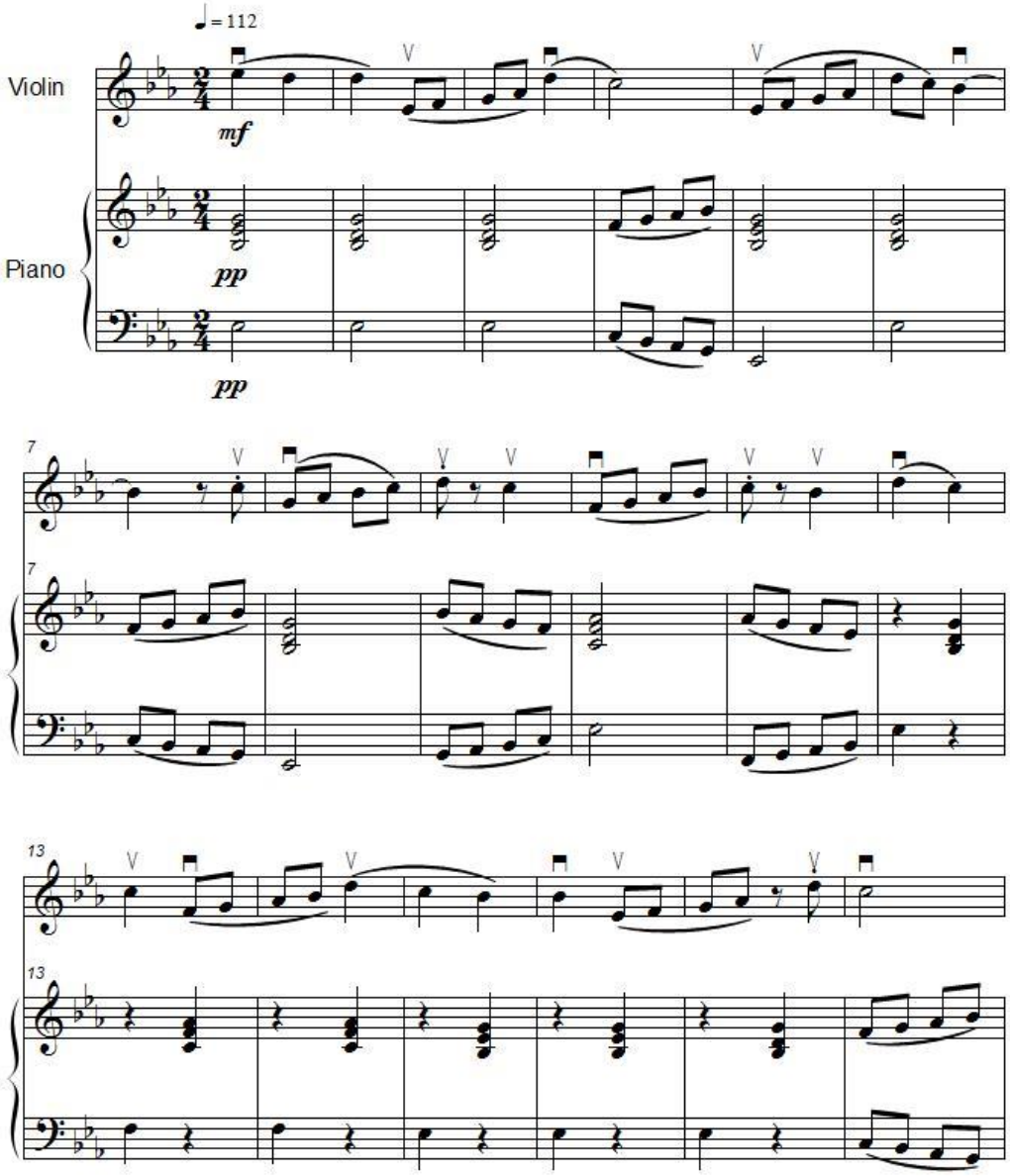
On the lake

by Boris Haase



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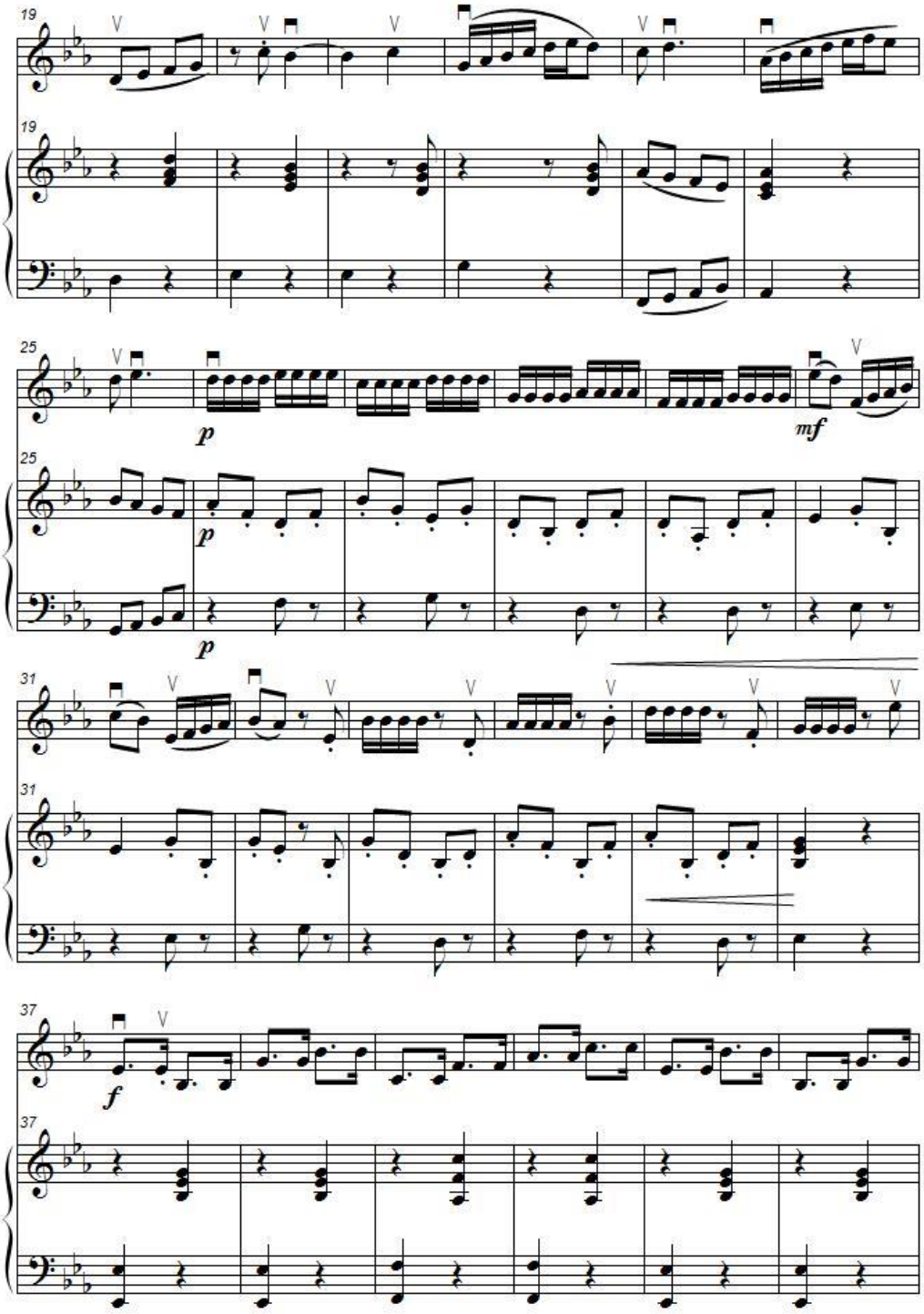
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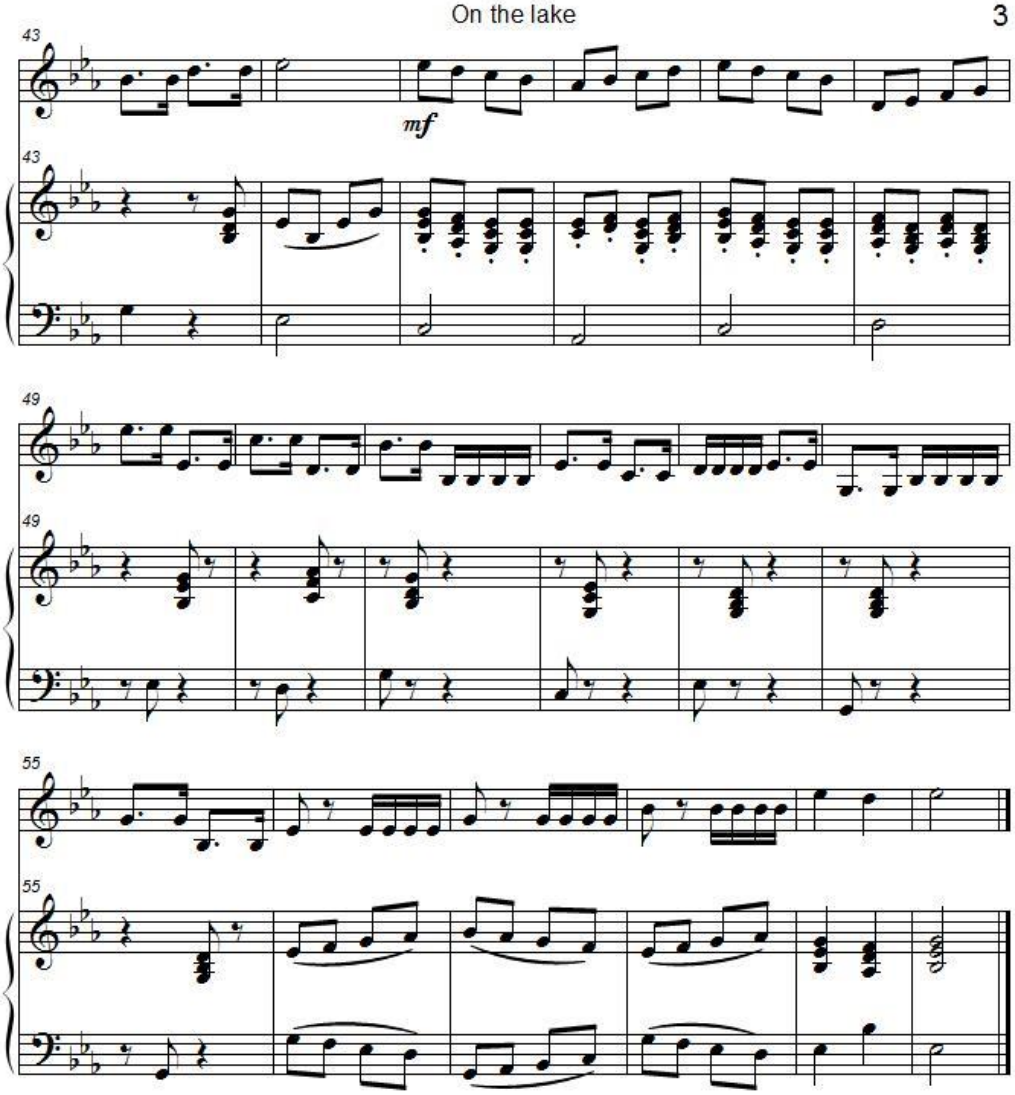
43
mf
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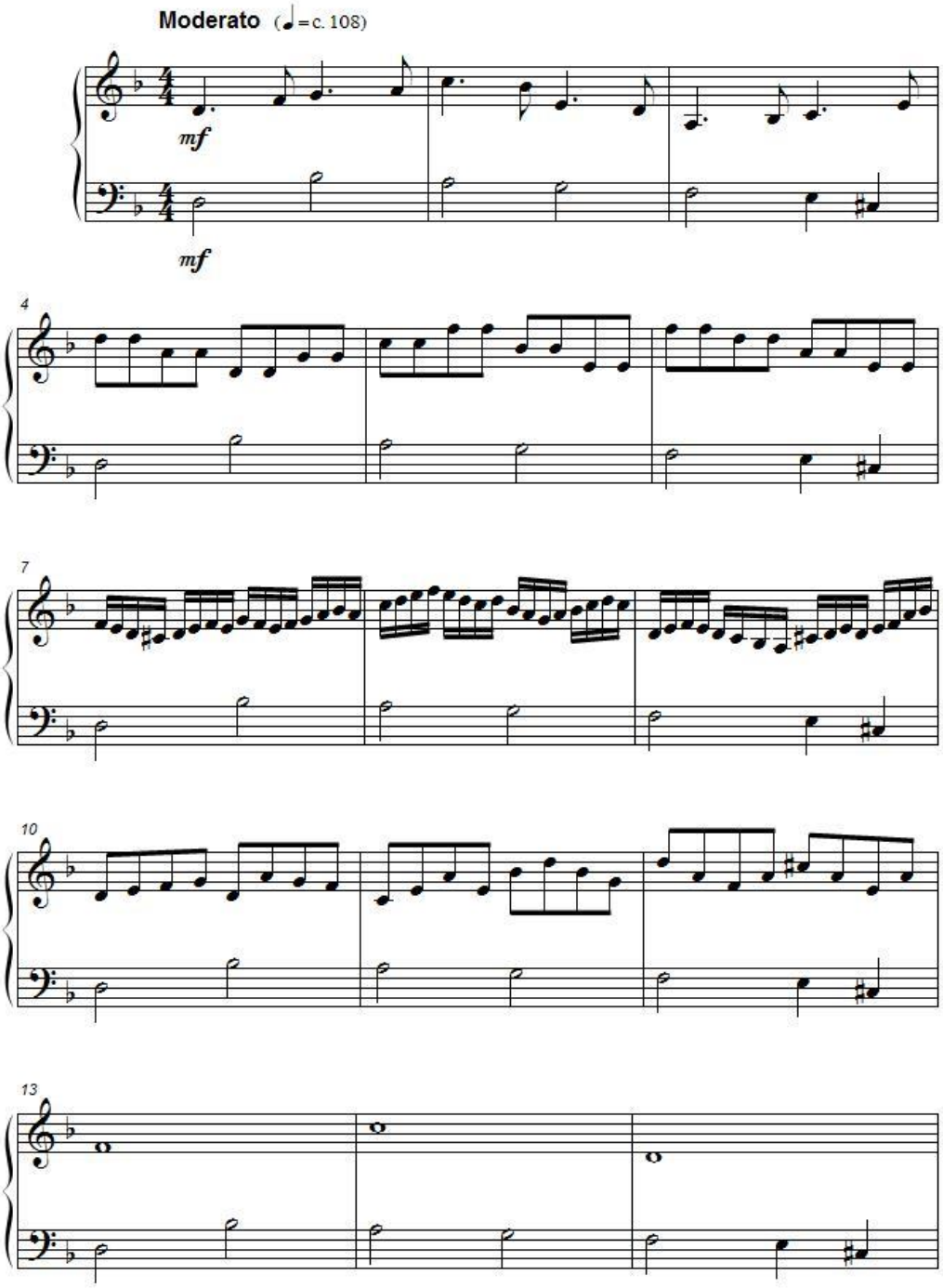
Beside the river

by Boris Haase



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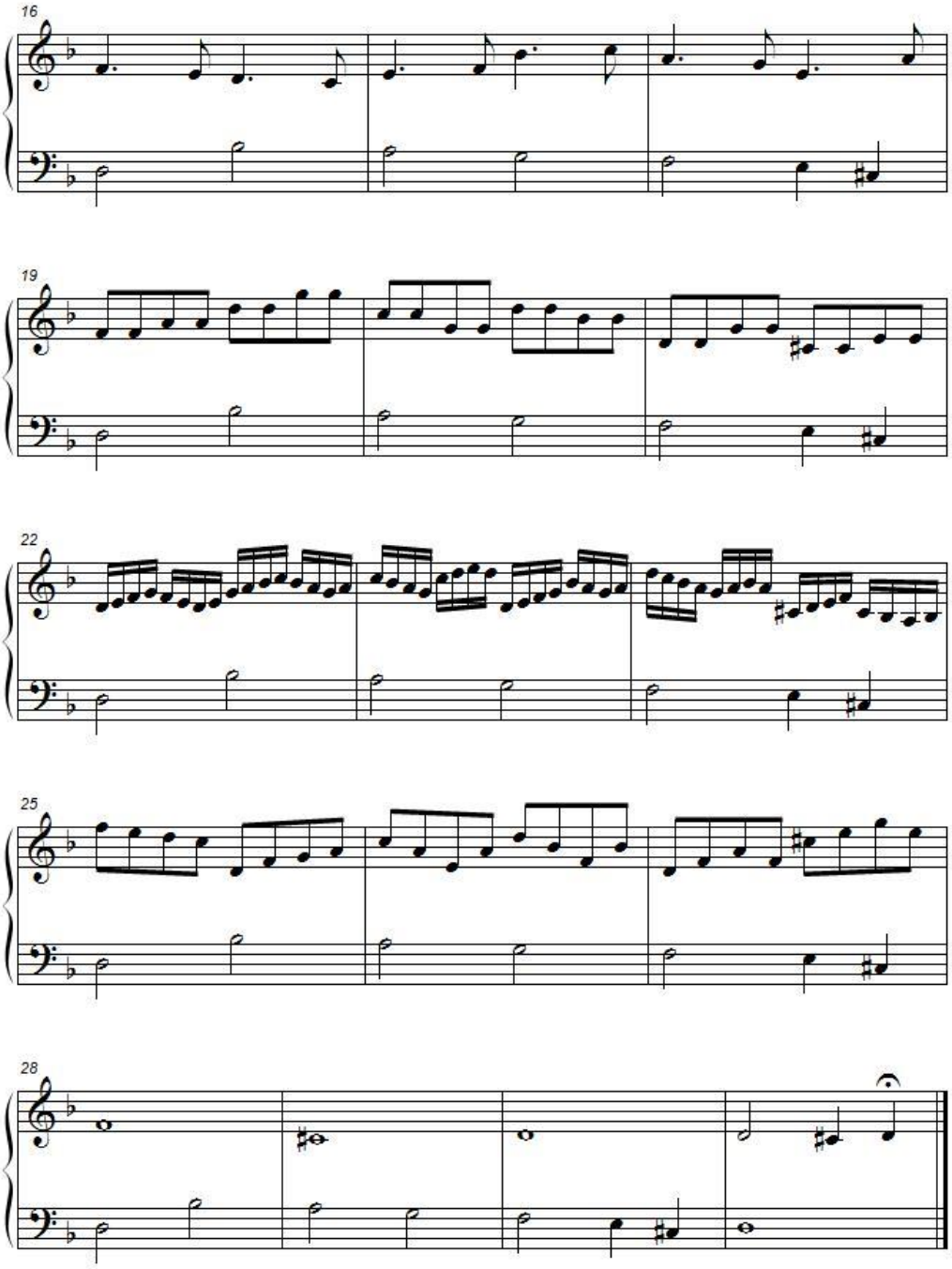
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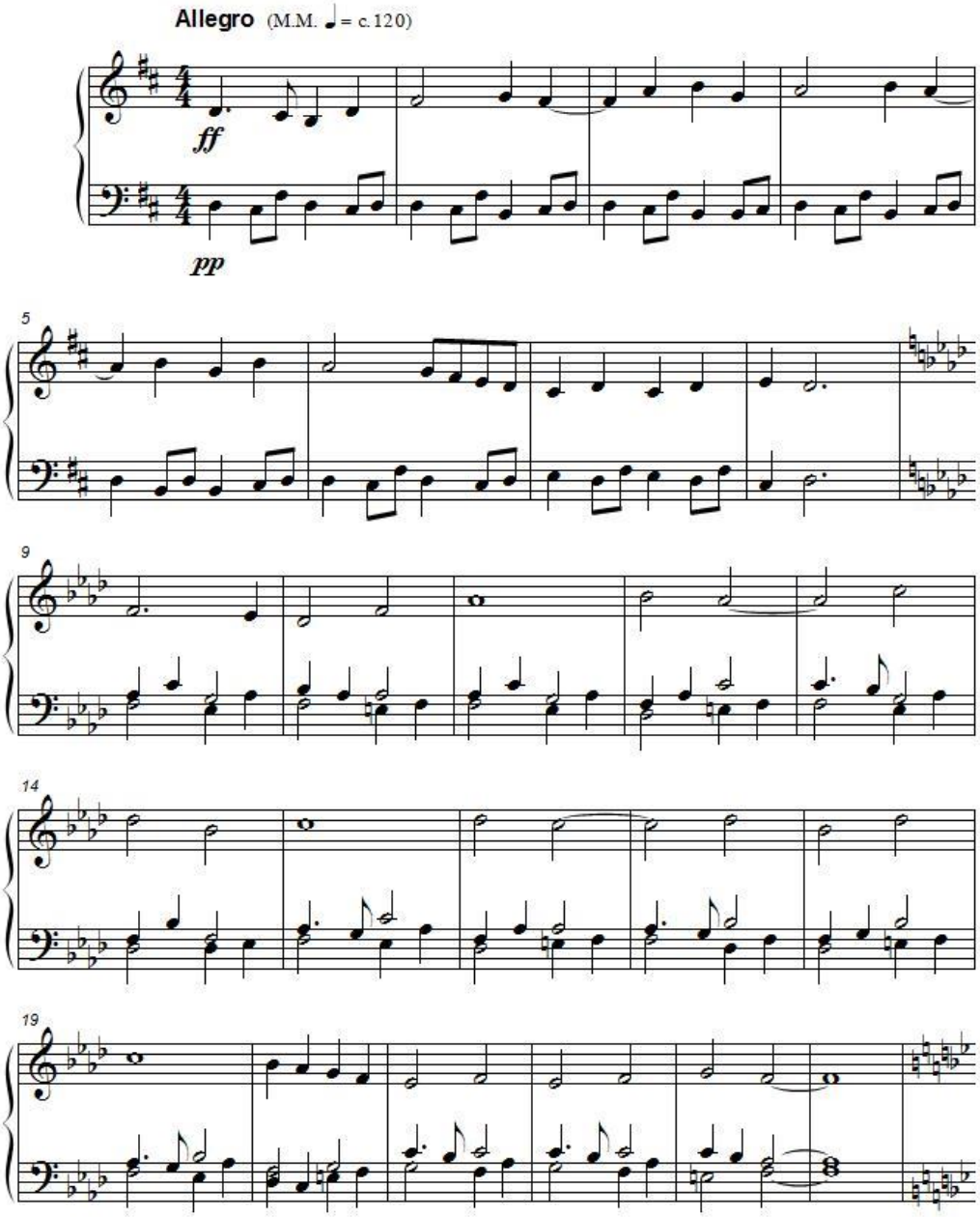
At the wall

by Boris Haase



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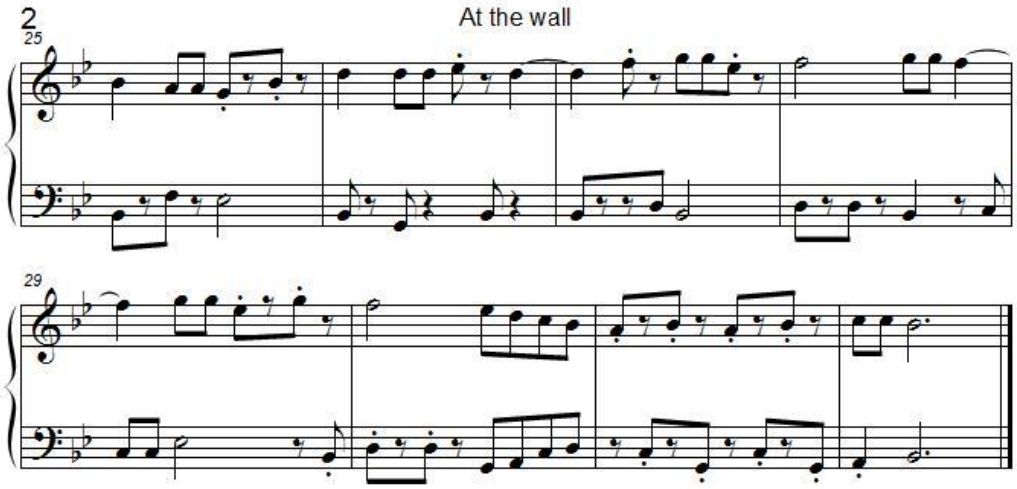
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At the wall
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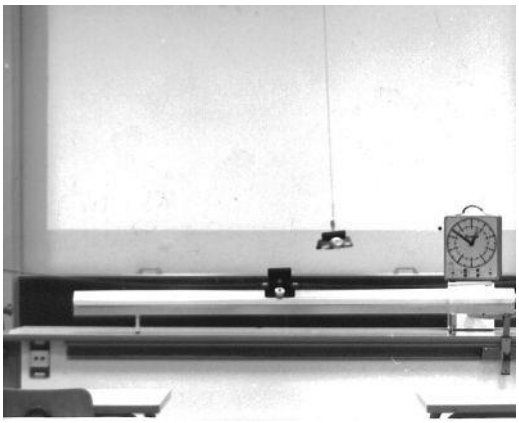
Physics

Below oscillations, as result of Jugend forscht 1983, an approximate formula, derived by differentiation, for the periodic time of selected oscillations of a mass point in the plane is deduced, which avoids the exact but cumbersome integration. The periodic time for the mathematical pendulum is experimentally checked. For the universe, the cube and the sphere model isomorphic to it are presented.

Oscillations



Youth researches 1983
Subject area physics

A novel method for the determination of the periodic time of the mathematical pendulum with large amplitudes and the discussion for the applicability on other selected oscillations of the plane

Compiled by Boris Haase (18),
Theodor-Heuss-Gymnasium in Göttingen

Abstract of the present paper

The oscillation period of mass points on curved oscillation ways in the vertical plane - like e.g. the mathematical pendulum and the cycloid pendulum -, is to be computed in the present paper in a simpler way than so far.

Most of these oscillation periods are - if the mass point on the oscillation way is far from the rest position, whose lowest point - dependent on this distance, which makes the computation generally more difficult.

In order to be able to compute an independent periodic time of this distance, the amplitude, we must know, except the mass, only the direction parameter, the quotient from repelling force and corresponding amplitude respectively direction way.

This knowledge is extended and applied in this paper to the oscillation period dependent on its amplitude.

Here partially differences to the conventional methods arise - by the example of the mathematical pendulum up to at most one per cent for 90° of amplitude – which is partially not the case as can be seen by the example of the cycloid pendulum, however. No references could be found for the ellipse pendulum, which has been also treated.

In the experimental part, the calculated periodic time could be confirmed for the mathematical pendulum considering the sources of error and the measuring accuracy.

The author concludes that the independently developed procedure could be well used in physics.

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1. Introduction

1.1. Reason and Basic Idea

The periodic time of the mathematical pendulum for large amplitudes was determined so far by quite difficult procedures of the infinitesimal calculus and series expansion. The new procedure developed in the present paper to determine the periodic time for selected oscillations of the plane is to simplify the calculation method on the one hand, on the other hand it is to compare with the old procedure for two examples regarding its accuracy.

Basic idea is the simultaneous simulation of the respective pendulum oscillation by a straight-lined harmonic oscillation (spring pendulum, oscillator).

1.2. Presuppositions

All selected oscillations emanate from a mass point, which is to swing in the vertical plane at a thread without mass around a fix point. The pendulum thread may be pulled to the side here - viewed from the rest position - at most by 90°, since the maximally repelling force must lie at initial height, in which no additional acceleration is to arise.

In order to prevent a free fall, otherwise bars would have to be used instead of threads, which could not, however, (as necessary) be clung to evolutes of the oscillation ways. Despite the clinging, the length of the pendulum is assumed in rest position for simplification reasons.

The radius of curvature of the oscillation way must remain either constant from the rest position or be continuously reduced, so that a free fall is prevented, too.

Friction and damping aspects are to be neglected.

Otherwise further computations are necessary for the periodic time, which would go, however, beyond the scope of this paper.

2. Main Part
2.1. Theoretical Solution of the Problem
2.1.1. General Solution

The periodic time of a harmonic oscillation is given by:

$$\text{(a) } T = 2\pi\sqrt{\frac{m}{D}}.$$

After the principle of conservation of energy, we have:

$$(b)\ ½\ m\ v^2 = ½\ D\ s^2 = m\ g\ h.$$

During an oscillation, the potential energy W_{pot} and the kinetic energy W_{kin} form the constant sum:

$$(c)\ W_{\text{pot}} + W_{\text{kin}} = W_{\text{pot}}{}^0.$$

$W_{\text{pot}}{}^0$ is to be expressed now by the tension energy W_{ten}, in order to determine a direction parameter $D(\varphi_0)$, dependent on the amplitude. $D(\varphi_0)$ is to be inserted then into the formula (a) to determine the periodic time.

For this purpose, an ersatz movement of the mass m is necessary by force F on the direction way s. We have:

$$(d)\ D = F/s \text{ (Hooke's law) and}$$

$$(e)\ F = ma = ml\ddot{\varphi} \text{ (Newton).}$$

Furthermore, we always resort to the direction parameter D of the plane thread pendulum for minimal amplitude. Here we have for the repelling and to the oscillation way tangential force F_R, since the normal force is compensated by the tension force of the thread (1, 2):

$$(f)\ F_R = mg \sin\varphi = -ml\ddot{\varphi}.$$

From this follows:

$$\text{(g) } \ddot{\varphi} = \sin\varphi = \frac{g}{l}.$$

For smallest amplitude angles φ, $\sin\varphi$ can be replaced by φ, and the conditions apply to the harmonic oscillation. Therefore, we have:

$$(h)\ \ddot{\varphi} = \omega^2\varphi$$

and from (g) follows:

$$\text{(i) } \omega = \sqrt{\frac{g}{l}}.$$

Thus, we have:

$$\text{(j) } D = m\,\omega^2 = m\frac{g}{l}.$$

For larger amplitudes the direction parameter D must consist of the product of the direction parameter for the harmonic oscillation in terms of a function value (φ_0) of the amplitude angle φ determining the inharmonic oscillation.

We write:

$$\text{(k) } D(\varphi_0) = \left(m\frac{g}{l}\right) f(\varphi_0).$$

The function value (φ_0), however, is not defined for the harmonic condition of the energy theorem, so that it must cancel if it is introduced. Writing $s = xl$ we have:

$$(l)\ \frac{1}{2}\frac{\frac{f(\varphi_0)mxg}{xl}(xl)^2}{\varphi_0} = mgh.$$

Hereby the not changing direction way s can be computed by the energy theorem, if one assigns the function value (φ_0) to the repelling force F_R. Writing

$$\frac{1}{2}\left(m\frac{g}{l}\right)s^2 = mgh$$

we have for the direction way:

$$(m)\ s = \sqrt{2lh}.$$

The repelling force F_R must be developed thus independently from the energy theorem. It is rather fixed by the initial oscillation as the tangentially attacking and thus maximally repelling force at initial height of the oscillation way. From

$$(n)\ F_R = (-)\ m\ a,$$

we obtain the periodic time with (a) and (d):

$$(o)\ T = 2\pi\sqrt{\frac{\sqrt{2lh}}{a}}.$$

Vividly, the direction way s represents with (m) the chord of a circle with the radius l. Since this circle is circle of curvature for the rest position of the oscillation way, this chord must lead from one point of the circular path at the initial height h to the rest position.

The mathematical pendulum (fig. 1) is the simplest form of an oscillation way, which aligns with the circle of curvature and possesses a punctiform evolute. Therefore, this is to be treated first.

2.1.2. The Mathematical Pendulum

From fig. 1, we obtain for the direction way s:

$$s = 2\sin\left(\frac{\varphi_0}{2}\right)l.$$

With respect to (f), we have for the maximally repelling force F_R:

$$F_R = (-)\ mg\sin\varphi_0.$$

The periodic time is regarding (a) and (d) therefore:

$$T = 2\pi\sqrt{\frac{l}{\cos\left(\frac{\varphi_0}{2}\right)g}}.$$

2.1.3. The Cycloid Pendulum

The common cycloid (fig. 2) has the parameter equation (2):

$$x = a\ (\phi - \sin\phi)\ \text{und}\ y = a\ (\cos\phi - 1).$$

We obtain for the radius of curvature ρ:

$$\rho = 4a \sin\left(\frac{\phi}{2}\right).$$

Since the angles ϕ increase from the initial height h to the rest position with $\phi = \pi$ or the sine decreases again from $\phi/2$ after exceeding the rest position towards the reversal point, the qualifications for the computation of the periodic time are fulfilled.

The initial height is:

$$h = 2\,a + y = a\,(\cos\phi_0 + 1).$$

If we plug h in (m), we have for $l = 4\,a$:

$$s = l\sqrt{\frac{\cos\phi_0 + 1}{2}}\cos\left(\frac{\phi_0}{2}\right)l.$$

Since the tangential acceleration b_t works always vertically to the radius of curvature ρ, we obtain:

$$b_t = \alpha\cos\left(\frac{\phi_0}{2}\right)g.$$

If $\phi_0 = 0$, then $b_t = g$, if $\phi_0 = \pi$, then $b_t = 0$. Therefore, the multiplier $\alpha = 1$ and we have:

$$b_t = \cos\left(\frac{\phi_0}{2}\right)g.$$

With respect to (o), the periodic time finally is:

$$T = 2\pi\sqrt{\frac{l}{g}}.$$

This was already proven by Huygens centuries ago.

2.1.4. The Ellipse Pendulum

The ellipse (fig. 3) has the parameter equation (3):

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

By differentiating the radius of curvature ρ we obtain (see appendix):

$$\rho = \frac{(a^4\text{-}a^2x^2 + b^2x^2)^{3/2}}{a^4b}.$$

From the rest position, where $x = 0$, x increases towards the reversal points, and since $a^2\,x^2 > b^2\,x^2$ holds, the radius of curvature is continuously reduced. We obtain thus for the rest position ρ:

$$\rho = l = \frac{a^2}{b}.$$

Writing

$$y = \text{-}b\sqrt{1\text{-}\frac{x^2}{a^2}}$$

the initial height h is:

$$h = b + y = b\left(1\text{-}\sqrt{1\text{-}\frac{x^2}{a^2}}\right).$$

Applying (m) we have thus for the direction way s:

$$s = \sqrt{2a\left(a\text{-}\sqrt{a^2\text{-}x^2}\right)}.$$

Hence, the direction way is independent of the semi-minor axis of the ellipse.

The tangential acceleration b_t is (after fig. 3):

$$b_t = \cos\beta\, g = \frac{bxg}{\sqrt{a^4\text{-}a^2x^2 + b^2x^2}}.$$

Applying (o) we obtain for the periodic time:

$$T = 2\pi\sqrt{\frac{\sqrt{a^4\text{-}a^2x^2 + b^2x^2}\sqrt{2a\left(a\text{-}\sqrt{a^2\text{-}x^2}\right)}}{bxg}}.$$

Although the periodic time is not defined for $x = 0$, we have for very small x, however:

$$T = 2\pi\sqrt{\frac{a^2}{bg}} = 2\pi\sqrt{\frac{l}{g}}.$$

For $x = a$, the proposition follows:

The periodic time on an ellipse with maximum amplitude is independent of the semi-minor axis.

2.2. Experiment to the Mathematical Pendulum

The experimental part was limited to the mathematical pendulum for temporal reasons.

Not only crafting evolutes would be otherwise very effortful, but also larger sources of error would be opened during the cycloid or ellipse oscillation. The difficulty to let the pendulum swing exactly to the evolute is an example for that, since the pendulum leaves the vertical plane already after few oscillations. Moreover, impulse or friction forces arise with the beginning of the swinging and in the bearing, which let the respective pendulum become the mathematical pendulum for large amplitudes.

The latter can be regarded therefore as part for the whole.

2.2.1. Experimental Setup

Two bars of a meter are connected to a third one and four sleeves and are pushed symmetrically through two ceiling hooks. In the centre, we fix additionally a protractor and the bearing for the pendulum thread of 1.60 m length.

An air track is placed in parallel to it on the bench such that its centre is vertically under the point of the protractor. A car of 0.2 kg mass placed on the centre is clamped by two springs such that we obtain a direction parameter of 2 N/m between the clamping at the end of the bench and the car.

A stopwatch is mounted ready to hand regarding the initial height of the pendulum and the car.

The experiment is illustrated on the title page. All devices used are specified also in the appendix.

2.2.2. Test Procedure and Results

First, we assured ourselves of the independence of the periodic time of the mass of the pendulum by attaching once a approx. 5 kg heavy ball, another time the car weighted on 1 kg of mass to the pendulum thread. The difference of the measured periodic time amounted to hundredths a second on each of the ten periods.

During the simulation of the mathematical pendulum by the harmonic oscillator, the time difference per oscillation amounted to 0.6 s (back and forth) with ten oscillations and 60° of maximum amplitude of the pendulum.

Computing the mass of the harmonic oscillator we used the formula:

$$m = \frac{2l}{\cos\left(\frac{\varphi_0}{2}\right) g}.$$

The latter can be determined directly from the periodic times for the two systems.

Here the direction parameter D_0 for the oscillator is twice as much the equal spring constants of the used tension springs. The periodic time of the oscillator is therefore:

$$T = 2\pi\sqrt{\frac{m}{2D}} \ (4).$$

The periodic time of the mathematical pendulum calculated above is:

$$T = 2\pi\sqrt{\frac{l}{\cos\left(\frac{\varphi_0}{2}\right) g}}.$$

The damping of the pendulum for amplitude 60° amounted to 1°, whereas the oscillator had one of 0.01 m for amplitude 0.9 m.

The appendix specifies the measured values for amplitudes of the mathematical pendulum from 0° to 90° in tab. 1.

2.3. Computations of errors

The periodic time of the mathematical pendulum was so far specified via the elliptical integral

$$\int_0^{\frac{\pi}{2}} \frac{d\phi}{\sqrt{1\text{-}k^2\sin^2\phi}}$$

where

$$k = \sin\left(\frac{\varphi_0}{2}\right)$$

giving (2):

$$T = 2\pi\sqrt{\frac{l}{g}\left(1 + \left(\frac{1}{2}\right)^2 k^2 + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 k^4 + \left(\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\right)^2 k^6 + \cdots\right)}.$$

To make a direct comparison with the new formula for the periodic time:

$$T = 2\pi\sqrt{\frac{l}{\cos\left(\frac{\varphi_0}{2}\right) g}}.$$

the function value

$$f(\varphi_0) = \sqrt{\frac{1}{\cos\left(\frac{\varphi_0}{2}\right)}}$$

is converted by means of the series where $|x| < 1$:

$$(1+x)^p = 1 + px + \frac{p(p-1)x^2}{1 \cdot 2} + \frac{p(p-1)(p-2)x^3}{1 \cdot 2 \cdot 3} + \cdots \quad (2)$$

and the relationship:

$$f(\varphi_0) = \frac{1}{\sqrt[4]{1\text{-}\sin^2\left(\frac{\varphi_0}{2}\right)}}$$

with respect to

$$x = \text{-}\sin^2\left(\frac{\varphi_0}{2}\right) = \text{-}k^2$$

into the comparison series:

$$1 + \left(\frac{k}{2}\right)^2 + \left(\frac{1 \cdot 5}{1 \cdot 2}\right)^2 \left(\frac{k}{2}\right)^4 + \left(\frac{1 \cdot 5 \cdot 9}{1 \cdot 2 \cdot 3}\right)^2 \left(\frac{k}{2}\right)^6 + \cdots$$

It follows that the two series are identical up to the first square member.

If we divide the old by the new series and subtract 1, we obtain the series of the relative error, where the former has a smaller modulus:

$$\frac{1}{64}k^4 + \frac{1}{64}k^6 + \frac{231}{16384}k^8 + \cdots.$$

If we divide conversely with identical subtraction, the relative error is larger:

$$\frac{1}{64}k^4 + \frac{1}{64}k^6 + \frac{235}{16384}k^8 + \cdots.$$

The number of the absolute error is the result of subtracting the old from the new series:

$$\frac{1}{64}k^4 + \frac{5}{256}k^6 + \frac{335}{16384}k^8 + \cdots.$$

The appendix records the values for selected angles from 0° to 90° of all series specified here in tables.

2.4. Discussion

Here the proportionalities arising from the formula are once to be considered:

$$T = 2\pi\sqrt{\frac{\sqrt{2lh}}{a}}.$$

First, it is to be noted that the direction way s is not proportional to the acceleration a of the repelling force F_R, but we must consider the function value (φ_0) contained in a.

The length of the oscillation way is adapted via the direction way:
If the initial height h is very small, the distance to be covered becomes smaller on the oscillation way in y-direction, and hence the periodic time, too.

If the oscillation way is moreover very flat, the small initial height is confronted by a large pendulum length, so that we obtain a mean for the length of the direction way, as it appears very clearly for maximum amplitudes of the ellipse pendulum.

For highly curved oscillation ways and large initial heights, the periodic time is relatively larger as with the mathematical pendulum.

In the direct proximity of the rest position, the numerical values of direction way and acceleration cancel out, so that the direction way becomes the length l of the pendulum and the acceleration becomes the standard acceleration g.

In the rest position itself, the periodic time is strictly speaking not defined, because the initial height h and the acceleration a become zero there.

The larger the acceleration a, the smaller the periodic time is, otherwise reverse conditions do exist.

These insights could be relative well confirmed in the experiment for the mathematical pendulum.

3. Conclusion

We can say in retrospect that the actual approach is not completely consistent with the one stated for clarity reasons.

Thus, first the problem of the mathematical pendulum was solved and then a general problem solution was established. All periodic times were computed without knowledge of the periodic times given in the corresponding literature, which was consulted several weeks later, when the experiment took place.

The calculation method could indeed be much simplified and the margin of error of 0.01 or one per cent needed not be exceeded regarding the accuracy for the old procedure.

Here the ellipse pendulum could not be included, since no references could be found about its periodic time.

The margin of error will most likely shift upwards, however, since the mathematical pendulum as special case of the ellipse pendulum such that its periodic time can be transferred for semi axes of equal length to the one of the mathematical pendulum, already deviates barely one per cent for maximum amplitude in relation to its periodic time.

The periodic time of the cycloid pendulum is, however, consistent with the references.

3.1. Critical Appreciation of the Paper

Without doubt, the problem of evolutes is not yet completely solved. If we do not presuppose anymore the pendulum length in rest position, we have perhaps again to resort to procedures of the infinitesimal calculus and series expansion, in order to obtain a higher precision.

On the one hand the mathematical pendulum, on the other hand the cycloid pendulum is contradictory to that.

For the first one, a distinct evolute is missing despite deviation, for the latter there is no deviation despite the presence of an evolute.

Thus, the new procedure must be considered as independent.

Measuring further periodic times than that of the mathematical pendulum is unfortunately missing in this paper and would clarify the circumstances more exactly. Here, however, very high requirements have to be put on the measuring accuracy and on the course of the experiments.

Altogether, the procedure developed in this paper requires a certain adjustment concerning the way of thinking; nevertheless, it should be applicable due to the good results obtained in physics.

4. Appendix

Figures

Abb. 1

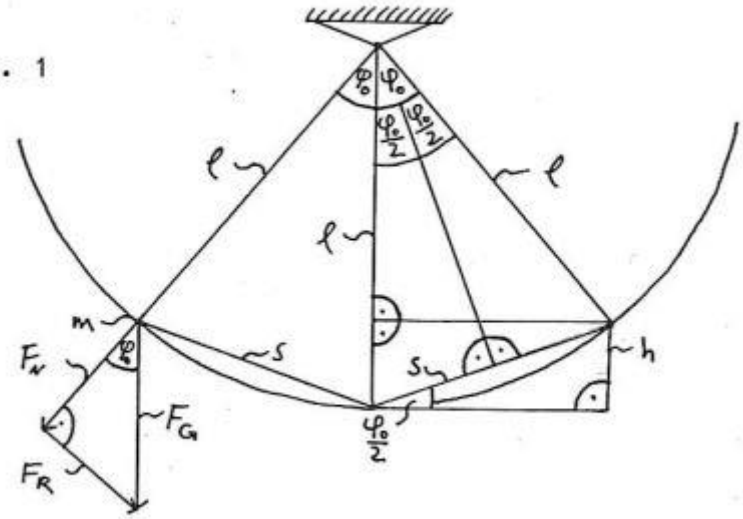


Abb. 2

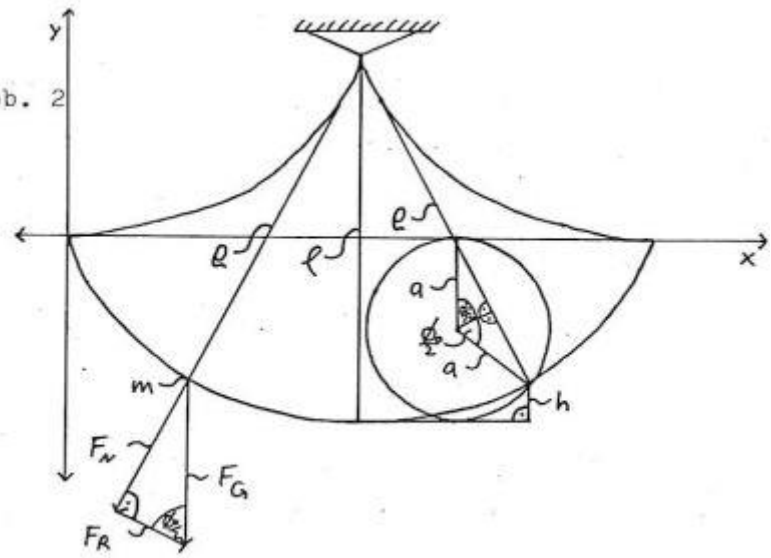
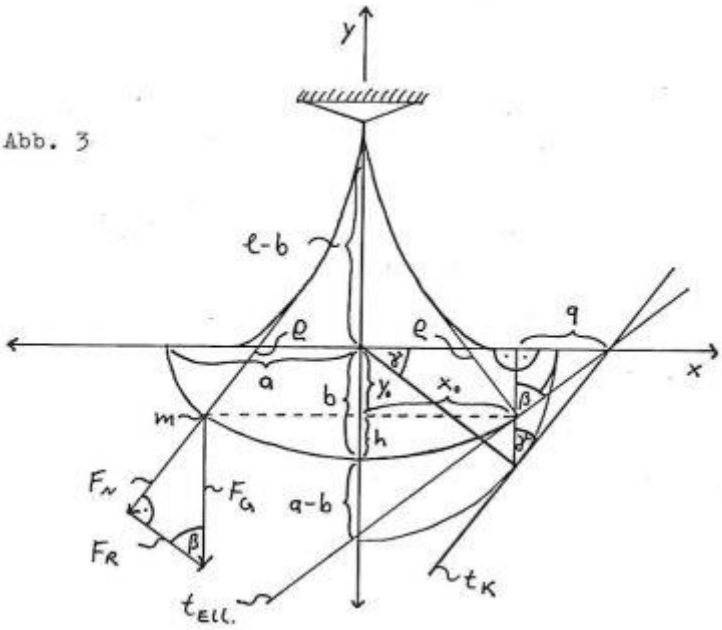


Abb. 3



All constructions took place with the aid of (9).

Auxiliary Calculations

The general formula (3) applies for the radius of curvature:

$$\rho = \frac{(\dot{x}^2 + \dot{y}^2)^{3/2}}{\dot{x}\ddot{y}\text{-}\dot{y}\ddot{x}}.$$

The common cycloid has the parameter equation (2):

$$x = a\,(\phi - \sin\phi) \text{ und } y = a\,(\cos\phi - 1).$$

For

$$(\dot{x}^2 + \dot{y}^2)^{3/2},$$

we can write therefore

$$(a^2\text{-}2\cos\phi\, a^2 + \cos^2\phi\, a^2 + \sin^2\phi\, a^2)^{3/2} = (2a^2\text{-}2\cos\phi\, a^2)^{3/2}$$

For

$$\dot{x}\ddot{y}\text{-}\dot{y}\ddot{x} = (a\text{-}\cos\phi\, a)(\text{-}\cos\phi\, a)\text{-}(\text{-}\sin\phi\, a)\sin\phi\, a = a^2\text{-}\cos\phi\, a^2,$$

we have thus for the radius of curvature ρ:

$$\rho = 2a\sqrt{2(1\text{-}\cos\phi)} = 4a\sin\left(\frac{\phi}{2}\right).$$

If we write for the ellipse temporarily $x = \sin\varphi\, a$ and

$$y = \text{-}b\sqrt{1\text{-}\frac{x^2}{a^2}} = \cos\varphi\, b,$$

we obtain:

$$\dot{x}\ddot{y}\text{-}\dot{y}\ddot{x} = \cos\varphi\, a(\cos\varphi\, b)\text{-}\sin\varphi\, b(\text{-}\sin\varphi\, a) = ab.$$

Thereby, the radius of curvature is:

$$\rho = \frac{(\cos^2\varphi a^2 + \sin^2\varphi b^2)^{3/2}}{ab}.$$

If we revoke the transformation, we obtain for ρ after expanding the fraction by a^3:

$$\rho = \frac{(a^4\text{-}a^2x^2 + b^2x^2)^{3/2}}{a^4b}.$$

The evolute of the common cycloid is also a cycloid with identical a (5).

For the evolute of the ellipse, we have, however, by (6):

$$\xi = \frac{(a^2\text{-}b^2)\cos^3 t}{a}.$$

and

$$\eta = \frac{(a^2\text{-}b^2)\sin^3 t}{b}$$

for $x = a\cos t$ and $y = b\sin t$.

The following initial considerations are necessary to compute the tangential acceleration b_t of the ellipse:

$$\tan\gamma = \frac{\sqrt{a^2\text{-}x^2}}{x} = \frac{q}{\sqrt{a^2\text{-}x^2}}.$$

From this follows directly:

$$q = \frac{a^2\text{-}x^2}{x}.$$

We can write for

$$\cos\beta = \frac{y}{\sqrt{q^2 + y^2}} = \frac{y}{\sqrt{\left(\frac{a^2\text{-}x^2}{x}\right)^2 + y^2}}$$

after expanding by

$$\frac{ax}{\sqrt{a^2\text{-}x^2}}$$

$$\cos\beta = \frac{bx}{\sqrt{a^4\text{-}a^2x^2 + b^2x^2}}.$$

To convert the formula for the periodic time of the ellipse pendulum:

$$T = 2\pi\sqrt{\frac{\sqrt{a^4\text{-}a^2x^2 + b^2x^2}\sqrt{2a\left(a\text{-}\sqrt{a^2\text{-}x^2}\right)}}{bxg}}.$$

into that of the mathematical pendulum, let $x = \sin(\varphi_0)\ a$ and $a = b = 1$. Thus, we have:

$$T = 2\pi\sqrt{\frac{a^2\sqrt{2\left(1\text{-}\sqrt{1\text{-}\frac{x^2}{a^2}}\right)}}{xg}} = 2\pi\sqrt{\frac{l\sqrt{2(1\text{-}\cos(\varphi_0)}}{\sin(\varphi_0)g}}.$$

Therefore, we finally obtain:

$$T = 2\pi\sqrt{\frac{l}{\cos\left(\frac{\varphi_0}{2}\right)g}}.$$

Test devices

2 table clamps for the bench with two holders for the springs
2 tension springs of 1 m length with a spring constant of approx. 2 N/m and a diameter of approx. 0.01 m
1 air track of 2 m length with air control
2 cars with blind and external hook of 0.2 kg each mass
1 sphere mass of approx. 5 kg
1 pendulum thread of at most 1.60 m length
1 stopwatch
2 ceiling hooks
1 up to 2 m extendable pillar with holder
7 sleeves
7 bars of twice 1 m, once 0.6 m, twice 0.5 m, once 0.1 m and once 0.2 m length as angle bars
1 clamp for protractor with small piece of wood
1 protractor
1 adhesive tape
as well as various loading weights

Tables

Tab. 1

$\varphi_0/°$	T_{exp}/s	T_{thn}/s	T_{tha}/s	$\Delta T_{an}/\%$	$\Delta T_{aa}/\%$
90	2.95	3.02	3.00	2.26	1.52
80	2.87	2.90	2.89	1.02	0.58
70	2.78	2.80	2.80	0.86	0.61
60	2.72	2.73	2.72	0.26	0.13
50	2.65	2.67	2.66	0.60	0.54
40	2.61	2.62	2.62	0.31	0.29
30	2.58	2.58	2.58	0.09	0.08
20	2.55	2.56	2.56	0.29	0.29
10	2.52	2.54	2.54	0.90	0.90
5	2.51	2.54	2.54	1.15	1.15

T_{exp} = experimentally determined periodic time (average values)

T_{thn} = theoretical periodic time by the new procedure

T_{tha} = theoretical periodic time by the old procedure

ΔT_{an} = absolute difference of T_{exp} and T_{thn} with T_{thn} as reference value

ΔT_{aa} = absolute difference of T_{exp} and T_{tha} with T_{tha} as reference value

Here, the constants are:

$l = 1.60$ m
$g = 9{,}80665$ m/s²

Tab. 2

$\phi_0/°$	$f(T_0)_n$	$f(T_0)_a$	$\Delta f(T_0)_{rkl}$	$\Delta f(T_0)_{rgr}$	$\Delta f(T_0)_a$
90	1.18921	1.18034	0.74558	0.75118	0.88665
80	1.14254	1.13749	0.44214	0.44411	0.50517
70	1.10489	1.10214	0.24815	0.24877	0.27418
60	1.07457	1.07318	0.12916	0.12933	0.13879
50	1.05042	1.04978	0.06045	0.06049	0.06350
40	1.03159	1.03134	0.02418	0.02418	0.02494
30	1.01748	1.01741	$8*10^{-3}$	$8*10^{-3}$	$8*10^{-3}$
20	1.00768	1.00766	$1*10^{-3}$	$1*10^{-3}$	$1*10^{-3}$
10	1.00191	1.00191	$1*10^{-4}$	$1*10^{-4}$	$1*10^{-4}$
5	1.00048	1.00048	$6*10^{-6}$	$6*10^{-6}$	$6*10^{-6}$
4	1.00030	1.00030	$3*10^{-6}$	$2*10^{-6}$	$2*10^{-6}$
3	1.00017	1.00017	$8*10^{-7}$	$7*10^{-7}$	$8*10^{-7}$
2	1.00008	1.00008	$3*10^{-7}$	$2*10^{-7}$	$2*10^{-7}$
1	1.00002	1.00002	$2*10^{-7}$	$1*10^{-7}$	$1*10^{-7}$
0	1.00000	1.00000	0	0	0

$f(T_0)_n$ = function value of the periodic time of the mathematical pendulum, by which the periodic time of the cycloid pendulums must be multiplied, by the new procedure

$f(T_0)_a$ = the same by the old procedure

$\Delta f(T_0)_{rkl}$ = smaller relative difference of the two

$\Delta f(T_0)_{rgr}$ = larger relative difference of the two

$\Delta f(T_0)_a$ = absolute difference of the two

Here, all delta $\Delta(T_0)_x$ are specified in per cent.

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Addendum:

Today, I am certainly in a position to calculate the correct periodic times and to state the error for my approximate formula.

We have:

$$mgh_0 = mgh + \frac{1}{2}m\frac{ds^2}{dt^2}.$$

For $x = (t)$ and $y = y(t)$, we obtain:

$$T = \sqrt{\frac{8}{g}}\int_0^{s_0}\sqrt{\frac{1}{h_0\text{-}h}}ds = \sqrt{\frac{8}{g}}\int_0^{x_0}\sqrt{\frac{1+y'^2}{y_0\text{-}y}}dx = \sqrt{\frac{8}{g}}\int_0^{t_0}\sqrt{\frac{\dot{x}^2+\dot{y}^2}{y_0\text{-}y}}dt.$$

The approximate formula reads:

$$T = 2\pi\sqrt{\frac{\sqrt{2lh}}{a}} = 2\pi\sqrt{\frac{\sqrt{2(1+y'(0)^2)^{3/2}y_0\left(1+y_0'^2\right)}}{gy_0'\sqrt{y''(0)}}} = 2\pi\sqrt{\frac{\sqrt{2(\dot{x}(0)^2+\dot{y}(0)^2)^{3/2}y_0(\dot{x}_0^2+\dot{y}_0^2)}}{g\dot{y}_0\sqrt{\dot{x}(0)\ddot{y}(0)\text{-}\dot{y}(0)\ddot{x}(0)}}}.$$

For $x = a\sin\varphi$, $y = b - b\cos\varphi$,

$$ds = \sqrt{dx^2+dy^2} = \sqrt{a^2\cos^2\varphi + b^2\sin^2\varphi}\,d\varphi, k = \sin\left(\frac{\varphi_0}{2}\right)$$

and

$$\varepsilon^2 = \frac{a^2\text{-}b^2}{a^2},$$

the approximate formula and $\varepsilon^2 < 1$ yield

$$T = \frac{2\pi a\sqrt[4]{1\text{-}\varepsilon^2\sin^2\varphi_0}}{\sqrt{bg\cos\left(\frac{\varphi_0}{2}\right)}}$$

and the series expansion:

$$T = \frac{2\pi a}{\sqrt{bg}}\left\{1+\frac{1}{4}k^2\text{-}\varepsilon^2k^2+\frac{5}{32}k^4+\frac{3}{4}\varepsilon^2k^4\text{-}\frac{3}{2}\varepsilon^4k^4+O(k^6)\right\}.$$

The ellipse pendulum implies

$$T = \sqrt{\frac{8a^2}{bg}}\int_0^{\varphi_0}\sqrt{\frac{1\text{-}\varepsilon^2\sin^2\varphi}{\cos\varphi\text{ - }\cos\varphi_0}}d\varphi,$$

giving the series expansion:

$$T = \frac{2\pi a}{\sqrt{bg}}\left\{1+\frac{1}{4}k^2\text{-}\varepsilon^2k^2+\frac{9}{64}k^4+\frac{3}{8}\varepsilon^2k^4\text{-}\frac{3}{4}\varepsilon^4k^4+O(k^6)\right\}.$$

The arithmetic-geometric mean $M\left(1,\cos\left(\frac{\varphi_0}{2}\right)\right)$ with quadratic convergence implies more precisely and for the mathematical pendulum (due to $a = b$ resp. $\varepsilon = 0$) exactly:

$$T = \frac{2\pi a}{\sqrt{bg}}\left\{\frac{1}{M\left(1,\cos\left(\frac{\varphi_0}{2}\right)\right)}\text{-}\varepsilon^2k^2+\frac{3}{8}(\varepsilon^2\text{-}2\varepsilon^4)k^4+\frac{5}{64}(\varepsilon^2+12\varepsilon^4\text{-}16\varepsilon^6)k^6+O(\varepsilon^2k^8)\right\}.$$

Universe

We can interpret the universe infinite n-dimensional as, for example, a cube, ball, or sphere. Here, the maximum extent can be finite, since an infinite subdivision is possible. The cube best reflects the homogeneity, the ball or the sphere does the uniform extension in each direction, without distinguishing certain directions from each other. Homogeneity and curvature need not be measurable.

The n dimensions cannot only include the space, but all finitely or infinitely scaled substances (see Reference Theory), so in particular the time. God can arrange the universe very differently. On a ball, for example, the simplest worlds may lie around the South Pole, and the most complex ones around the North Pole. By way of illustration, a suitable generalisation of the Riemann number sphere may serve.

We may be able to determine the size of the radius in our world, but not in general. Smallest units of a substance like the Planck units may not exist in some world. The concept of infinity is not fixed on time, but can be extended to any (measurable) substance via the concepts of unity and scale. However, the successive measuring of a unit on a scale predestines the concept of time.

Spatial points may not have any form or extension, so they can be pushed together at will. They are nevertheless something and may have gaps with nothing in between. It is our intuition that can fill in the intervening and give it expansion. The same applies to all other substances: Different laws or (lawless) freedoms may be applied to the same substances.

Their assignments to us result in different experience or parallel worlds for us. This leads to a so-called penetration model. It is important that God makes the (nature) laws appropriate. This is especially true for chaotic or highly inhomogeneous worlds and those with multiple timelines. References help to overcome endless distances. Every creature has deserved a world in the universe according to its karma.

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Advice — Advice
Amplitude — Oscillations

B

Bit sort — Theoretical Informatics
Books — Bibliography

C

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Classification — Equitable Distributing
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D

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E

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I

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K

Kinetic energy — Physics

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Number	Set Theory
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Definitions and Glossary

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C

Cuboid scheme: assignment of numbers to the set of natural numbers per dimension.

E

Earthday: working day inserted between Sunday and Monday.

I

ID: identification number assigned uniquely to every person.

L

Label: referred to of a distinguished.
Lept: eighth part of an oct, 64th part of a day.

N

Number: selecting marking of the many and construction from it.

O

Oct: eighth part of a day.
Orbital: meaning environment of a term.

P

Pfeil: future programming language.

S

Signature: electronic equivalent to the manual signature.
Strap: multifunction device with diverse functions of the new media.

U

URL, generic: URL which contains wildcards.

W

World: closed total which is accessible for a subject during a period.

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